College of Engineering

Hydraulic Structures & Water Resources Department

Fluid Mechanics Lectures

for

Second Stage

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References :

1. Elementary Fluid Mechanics by J. K. Vennard & R.L. Street
2. Schaum Series
3. Mechanics of Fluids by Bernard Massey
4. Fluid Mechanics by Frank M. White
5. ميكانيك الموائع ، تأليف نعمة حمد عمارة

Main Objects

First Semester :

1. Introduction (Fluid Properties)
2. Measurement Methods for Fluid Pressures
3. Hydrostatics Forces
4. Accelerated fluids with linear and angular acceleration
5. Types of Flow , Fluid Flow
6. Continuity Equation (Mass Conservation)
7. Energy Conservation Law (Bernoulli's Equation)
8. Momentum Equation

Second Semester :

9. Real Flow in closed Pipes, Friction Coefficient & Minor Losses
10. Pipes in Succession
11. Pipes in Parallel
12. Pipes Network
13. Unsteady Flow
14. Introduction to Flow in Open Channel
15. Dimensional Analysis

Chapter One

INTRODUCTION

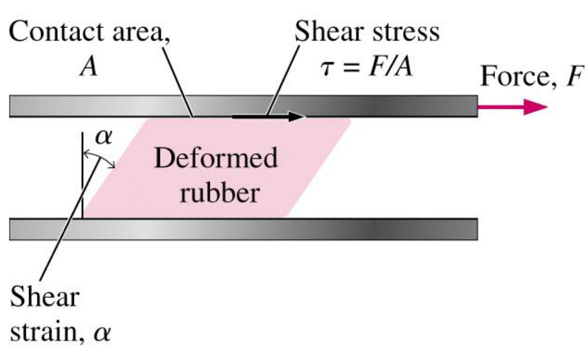
FLUID PROPERTIES

1.1. *Definitions* :

A **fluid** is defined as a substance that deforms continuously whilst acted upon by any force tangential to the area on which it acts. Such a force is termed a *shear force*, and the ratio of the shear force to the area on which it acts is known as the *shear stress*.

Fluid Mechanics: Fluid mechanics may be defined as that branch of engineering science that deals with the behavior of fluid under the condition of rest and motion
Fluid mechanics may be divided into three parts: Statics, Kinematics, and Dynamics

. **Statics** Deals with fluid at rest in equilibrium state, no force no acceleration.
. **Kinematics** Deals With flow behaviors of fluid like velocity, acceleration and flow patterns.
. **Dynamics** Deals with the effects of flow behaviors on fluid surroundings like forces and momentum exchange.



1.2. Fluid Properties :

1. **Mass Density (ρ) (Specific Mass) :** the ratio of the mass of a given amount of the substance to the volume it occupies. For liquids, this definition is generally satisfactory

$$\rho = \frac{m}{v} \quad (kg/m^3) \;.... \;(1.1)$$

where m : mass of the fluid (kg) , v : volume (m^3)

Mean Density : the ratio of the mass of a given amount of a substance to the volume that this amount occupies. If the mean density in all parts of a substance is the same then the density is said to be *uniform.*

Relative Density: is the ratio of the density of a substance to some standard *Relative density* density.

2. **Weight Density (γ) (Specific Weight) :** is defined as the weight per unit volume at the standard temperature and pressure,

$$\gamma = \frac{w}{v} = \rho * g \quad (N/m^3) \;.... \;(1.2)$$

where w : weight of the fluid (kg) , v : volume (m^3)

g : gravitational acceleration = 9.81 m/s^2

γ of water = 9810 N/m^3 at 4°C and 1 Bar

3. **Specific Volume (v):** It is defined as a volume per unit mass of fluid,

$$v = \frac{V}{m} = \frac{1}{\rho} \;.... \;(1.3)$$

Its unit are m^3/Kg.

4. **Specific Gravity (Sg):** It is defined as the ratio of the specific weight of the fluid to the specific weight of a standard fluid

For liquids the standard fluid is pure water at 4°C,

$$Sg = \frac{\gamma_{liquid}}{\gamma_{water}} \;.... \;(1.4)$$

For Gasses the standard fluid is air

***Example (1)*: Calculate the Specific weight, specific mass, specific volume and specific gravity of a liquid having a volume of 6m^3 and weight of 44 kN.**

Solution:

W=44 kN , V= 6 m3

$$\gamma = \frac{W}{V} = \frac{44}{6} = 7.333\ kN/m^3$$

$$\rho = \frac{\gamma}{g} = \frac{7.333 \times 1000}{9.81} = 747.5\ kg/m^3$$

$$v = \frac{1}{\rho} = \frac{1}{747.5} = 0.00134\ m^3/kg$$

$$Sg = \frac{\gamma_{liquid}}{\gamma_{water}} = \frac{7.333}{9.81} = 0.747$$

***Example (2)*: Calculate the Specific weight, specific mass, specific volume and density of chlorine gas(cl2) at 80^{0}F and pressure of 100 psia(Ibs per square inch, absolute)**

Solution:

Atom. Wt. chlorine = 35.45 mol. Wt. = 2*35.45 = 70.9

R = 49709/70.9 = 701.1 ft . lb / slug ^{o}R

γ = 32.2 * 100 * 144 / 701.1 * 540 = 1.225 lb/ft^3

specific volume = 1/1.225 = 0.815 ft^3/lb

ρ = 1.225/32.2 = 0.0381 slug/ft^3

***Example (3)*: Calculate the Specific weight, specific volume and density of carbon dioxide gas(CO_2) at 100^{0}C and atmospheric pressure of 101.3 kN/m^2(kilo Newton per square meter, absolute)**

Solution:

Atom. Wt. carbon = 12 , Atom. Wt. oxygen = 16 mol. Wt. = 2*16+12 = 44

R = 8313/44 = 189 ft . lb / slug ^{o}R

γ = 9.81 * 101.3 * 10^3/ 189 * 373 = 1.225 lb/ft^3

specific volume = 1/14.1 = 0.071 m^3/N

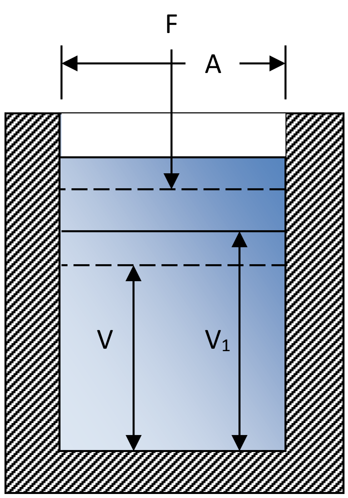
$\rho = \gamma/g$ = 14.1/9.81 = 1.44 kg/m^3

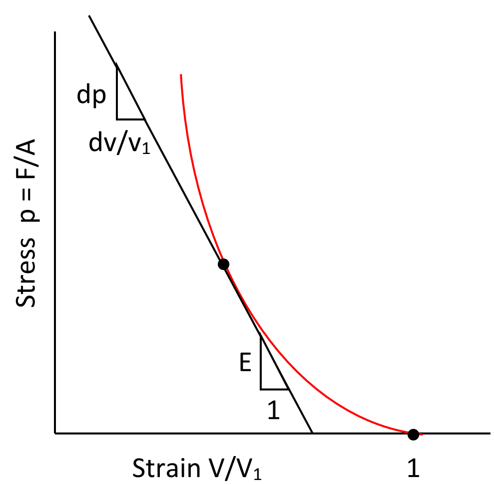
5. Elasticity (Compressibility) :

All matter is to some extent compressible. That is to say, a change in the pressure applied to a certain amount of a substance always produces some change in its volume. Although the compressibility of different substances varies widely, the *proportionate* change in volume of a particular material that does not change its phase (e.g. from liquid to solid) during the compression is directly related to the change in the pressure.

Bulk modulus of The degree of compressibility of a substance is characterized by the *modulus of elasticity*, *E*, which is defined by the equation:

$$E = -\frac{dp}{dv/v_1} \quad ....(1.5)$$





There is small pressure disturbance, that travels as a wave of increased (or decreased) density and pressure, moves at a celerity, a , given by :

$$a = \sqrt{\frac{E}{\rho}} \quad .... (1.6)$$

Where a is frequently called the sonic or acoustic velocity because sound, a small pressure disturbance, travels at this velocity.

Example (4) *: **Air at 60° F and 14.7 psia is compressed isentropically so that its volume is reduced 50%. Calculate the final pressure and temperature and the sonic velocities before and after compression.***

Solution:

$\gamma_1 = 32.2 * 14.7 * 144 / 1715 * 520 = 0.0763\ lb/ft^3$

$\gamma_2 = 2 * 0.0763 = 0.1526\ lb/ft^3$

$$\frac{p_2 * 144}{(0.1526)^{1.4}} = \frac{14.7 * 144}{(0.0763)^{1.4}}$$

$P_2 = 38.8$ psia

$$\frac{32.2 * 38.8 * 144}{1715 * T_2} = 0.1526$$

$T_2 = 687°\ R\ (227°\ F)$

$a_1 = \sqrt{1.4 * 1715 * 520} = 49\sqrt{520} = 1118$ fps

$a_2 = 49\sqrt{687} = 1285$ fps

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6. Viscosity :

The most important of fluid properties is viscosity, which relates the local stresses in a moving fluid to the strain rate of the fluid element. When a fluid is sheared, it begins to move at a strain rate inversely proportional to a property called its *coefficient of viscosity*. Consider a fluid element sheared in one plane by a single shear stress, as in figure below. The shear strain angle will continuously grow with time as long as the stress is maintained, the upper surface moving at speed *u* larger than the lower. Such common fluids as water, oil, and air show a linear relation between applied shear and resulting strain rate

$$\tau\ \alpha \frac{d\theta}{dt} \quad .... (1.7)$$

from the geometry of figure below, we see that:

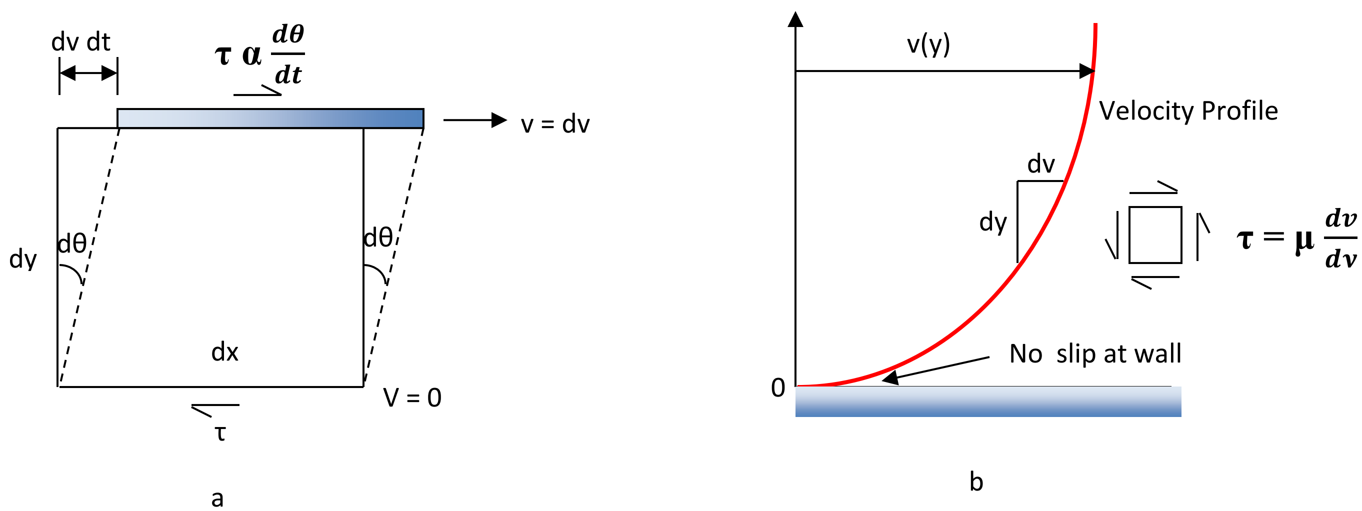
$$\tan d\theta = \frac{dv\ dt}{dy} \quad ....(1.8)$$

In the limit of infinitesimal changes, this becomes a relation between shear strain rate and velocity gradient :

$$\frac{d\theta}{dt} = \frac{dv}{dy} \quad ....(1.9)$$

From equation (1.7), then, the applied shear is also proportional to the velocity gradient for the common linear fluids. The constant of proportionality is the viscosity coefficient , μ :

$$\tau = \mu \frac{d\theta}{dt} = \mu \frac{dv}{dy} \quad .... (1.10)$$



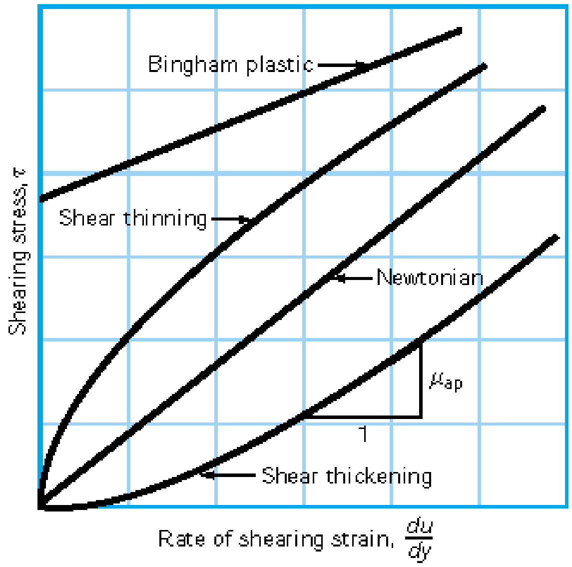
Therefore, the coefficient of **dynamic viscosity**:

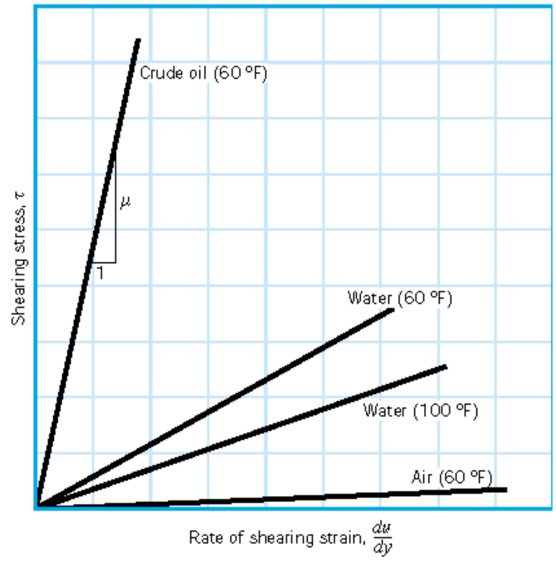
$$\mu = \frac{\tau}{\frac{du}{dy}} \quad .... (1.11)$$ **(N. s/m^2 or Pa .s or kg/m .s)**

***Kinematic Viscosity* ,** ν = the ratio of dynamic viscosity to mass density

$$\nu = \frac{\mu}{\rho} \quad .... (1.12)$$ **(m^2/s)**

The unit Stoke (St) is also used where $1^{St} = 10^{-4}$ m^2/s (1^{St} = cm^2/s)





- The fluid is **non-Newtonian** if the relation between shear stress and shear strain rate is **non-linear**
- Typically, as temperature increases, the viscosity will decrease for a liquid, but will increase for a gas.

***Example (5)* : Oil has dynamic viscosity (μ = 1.0 × 10-3 Pa.s) filled the space between two concentric cylinders, where the inner one is movable and the outer is fixed. If the inner and outer cylinders has diameters 150mm and 156mm respectively and the height of both cylinders is 250mm, determine the value of the torque (T) that necessary to rotate the internal cylinder with 12 rpm?**

Solution:

$$v = \frac{rpm}{60} \times 2\pi r = \frac{12}{60} \times 2\pi \times 0.075 = 0.09425 \text{ m/s}$$

$$T = \tau Chr$$

$$\tau = T(2\pi r \times h \times r)^{-1} = T(2\pi hr^2)^{-1}$$

$$\tau = \mu \frac{dv}{dy} = \mu \frac{\Delta v}{\Delta y}$$

$$\tau = 10^{-3} \times \frac{0.09425}{0.003} = 31.41667 \times 10^{-2} \frac{N}{m^2}$$

$$T = 31.41667 \times 10^{-2} \times 2 \times .075 \times \pi \times .25 \times .075 = 2.7 \times 10^{-4} \, N.m$$

Example (6) : ***Oil has a density of 580 kg/m^3 flow through a pipe its diameter 200 mm. If it is known from the pressure calculations for a certain length of the pipe that the shear stress at the pipe wall equal to 0.07 N/m^2, and its known from the velocity calculations through a certain cross section of the pipe that the velocity profile equation is :***

$V = 1\text{-}100r^2$

Where the velocity dimension is in m/s and the distance from the centre of pipe r in m. If the flow is laminar, calculate the kinematic viscosity for the oil?

Solution:

For laminar flow :

$$\tau = \mu \frac{dv}{dy}$$

the distance from the centre of pipe is

$$r = R - y$$

Where R is the radius of the pipe, y is the distance from the pipe wall toward the pipe centre.

Where $dr = -dy$ and then the above equation become:

$$\tau = -\mu \frac{dv}{dr}$$

And from the section of velocity distribution, the strain in any point equal to :

$$\frac{dv}{dr} = -200r$$

then the shear stress in any distance from the pipe centre is expressed as :

$$\tau = 200\mu r$$

whereas τ at the wall equal to 0.07 N/m^2 then by substitution this value in the last equation we obtain follows:

$$\mu = \frac{0.07}{20} \frac{N.s}{m^2} = 0.0035 \text{Pa.s}$$

and the kinematic viscosity is :

$$\nu = \frac{\mu}{\rho} = \frac{0.0035 N.s.m^{-2}}{850 kg.m^{-3}} = 4.1176 \times 10^{-8} \frac{N.s.m}{kg} = 4.1176 \times 10^{-8} \frac{m^2}{s}$$

***Example (7)*: The velocity distribution for flow over a plate is given by u= 2y + y^2 where u is the velocity in m/s at a distance y meters above the plate surface. Determine the velocity gradient and shear stresses at the boundary and 1.5m from it. Take dynamic viscosity of fluid as 0.9 N.s/m^2**

Solution:

Given u= 2y + y^2 $\therefore \frac{du}{dy} = 2 - 2y$

1) *Velocity gradient,* $\frac{du}{dy}$

   At boundary :at y=0 $\frac{du}{dy} = 2\ s^{-1}$

   At y=0.15m $\frac{du}{dy} = 2 - 2 \times 0.15 = 1.7 s^{-1}$

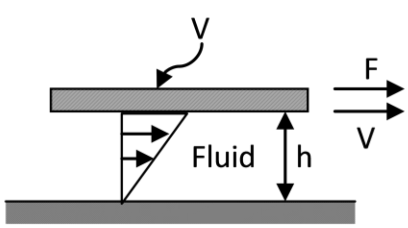
2) *Shear stresses, τ:*

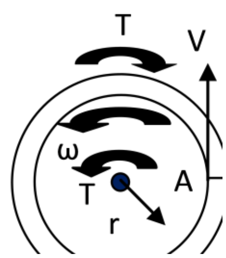
$$\tau)_{y=0} = \mu \frac{du}{dy} = 0.9 \times 2 = 1.8 \frac{N}{m^2}$$

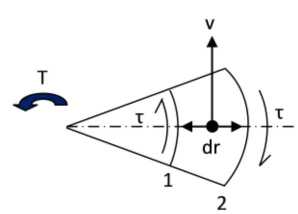
$$\tau)_{y=0.15} = \mu \frac{du}{dy} = 0.9 \times 1.7 = 1.53\ N/m^2$$

***Example (8)*: A cylinder 0.28 ft. in radius and 2 ft. in length rotates coaxially inside a fixed cylinder of the same length and 0.3 ft radius. Glycerin (μ = 0.031 lb.sec/ft²)fills the space between the cylinders. A torque of 0.5 ft.lb is applied to the inner cylinder. After constant velocity is attained, calculate the velocity gradient at the cylinder walls, the resulting rpm, and the power dissipated by fluid resistance. Ignore end effects.**

Solution:







Refer to figure 2 above for a definition sketch. The torque T is transmitted from inner cylinder to outer cylinder through the fluid layers; therefore (r is the radial distance to any fluid layer and *l* is the length of the cylinders)

T = - τ (2 π r * *l*) r ; $\tau = -\frac{T}{2\pi r^2\, l} = \frac{\mu\, r\, d\,(\frac{v}{r})}{dr}$

Consequently :

$$\frac{d\left(\frac{v}{r}\right)}{dr} = -\frac{T}{2\pi\,\mu\, l\, r^3}$$

and

$$\int_{\frac{v}{r_i}}^{0} d\left(\frac{v}{r}\right) = \frac{-T}{2\pi\mu\, l}\int_{r_i}^{r_o}\frac{dr}{r^3} \quad ; \quad -\frac{v}{r_i} = \frac{T}{4\pi\,\mu\, l}\left[\frac{1}{r_o^2} - \frac{1}{r_i^2}\right]$$

$$V = \frac{-0.28*0.5}{4\pi*0.031*2}\left[\frac{1}{(0.3)^2} - \frac{1}{(0.28)^2}\right] = 0.296 \text{ ft/sec}$$

The power dissipated in fluid friction is :

P = 2 π r_i τ_i V*l* = Tω

Where ω = V / r_i = 1.06 sec^{-1} is the rotational speed (radians/sec)

Numerically :

$$\left(\frac{dv}{dr}\right)_i = \frac{-T}{2\pi\mu\, l\, r_i^2} + \frac{v_i}{r_i} = -16.4 + 1.1 = -15.3 \text{ ft / sec / ft}$$

$$\left(\frac{dv}{dr}\right)_o = \frac{-T}{2\pi\mu\, l\, r_o^2} + \frac{v_o}{r_o} = -14.2 + 0 = -14.2 \text{ ft / sec / ft}$$

rpm = $\left(\frac{\omega}{2\pi}\right) * 60 = 10.1$

P = 0.5 * 1.06 = 0.53 ft.lb/sec = 0.00096 hp

This power will appear as heat, tending to raise the fluid temperature and decrease its viscosity; evidently a suitable heat exchanger would be needed to preserve the steady state conditions given.

When h = $r_o - r_i$ $\longrightarrow$ 0 , then h / r_i is small and dv/dr $\longrightarrow$ - V / h

As a consequence, v/r is much less than dv/dr :

$\tau \approx \mu \frac{dv}{dr} = -\frac{\mu V}{h}$

Assuming this linear profile case for an approximate calculation gives :

h = 0.02 ; $\frac{h}{r_i} \approx 0.07$

$V = \frac{T}{2\pi\mu\, l}\left(\frac{h}{r_i^2}\right) = 1.28\ \frac{0.02}{0.0784} = 0.327$ ft / sec. = 11.2 rpm

Because these results differ from the former by almost 11%, the approximation is not satisfactory in this case.

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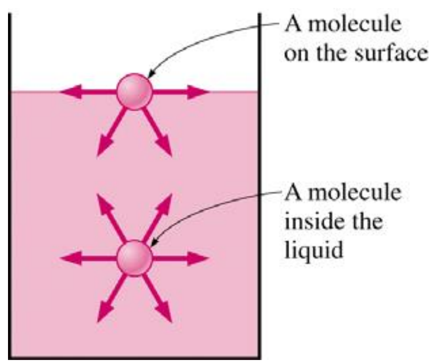
7. Surface Tension (Capillarity) :

The apparent tension effects that occur on the surfaces of liquids, when the surfaces are in contact with another fluids or a solid, depend fundamentally upon the relative sizes of intermolecular cohesive and adhesive forces. Although such forces are negligible in many engineering problems, they may be predominant in some such as:

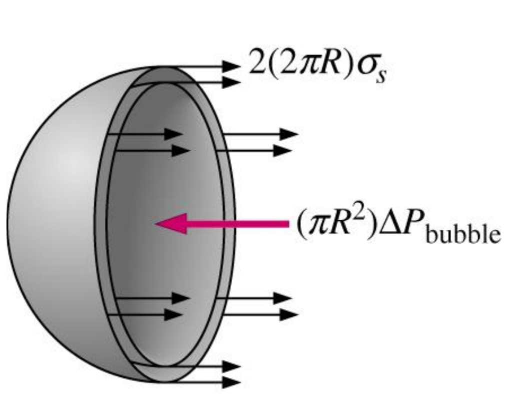
1. The capillary rise of liquids in narrow spaces.
2. The mechanics of bubble formation.
3. The breakup of liquid jets.



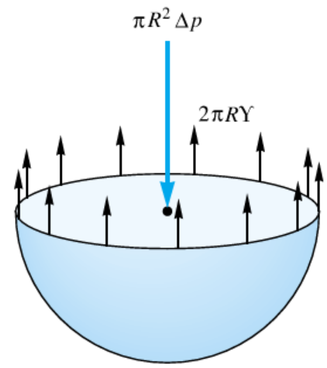
Surface tension is a properties of certain fluid-fluid interface

water-air ….. 0.075 N/m at 20^{o}C Water-air …. 0.056 N/m at 100^{o}C

mercury- air … 0.1 N/m



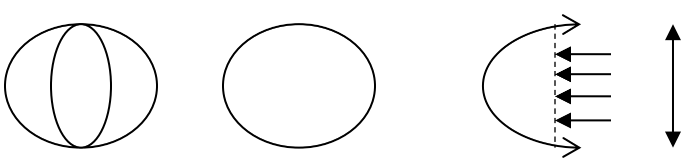
(b) Half a bubble



pressure inside water droplet:

let P= The pressure inside the drop
d= Diameter of droplet
σ= Surface tension of the liquid (water-air interface)

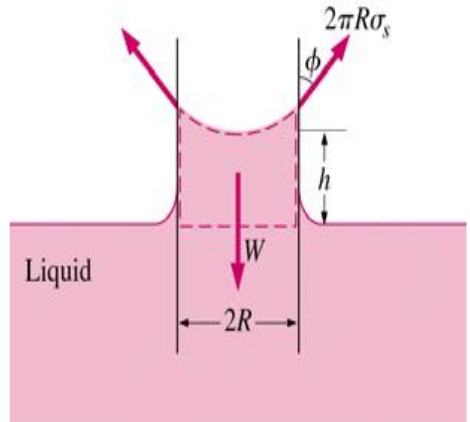
From sectional free body diagram of water droplet we have



1. ΔP between inside and outside = P-0 =P
2. Pressure force $=P \times \frac{\pi}{4} d^2, and$
3. Surface tension force acting around the circumference= $\sigma \times \pi d,$
   under equilibrium condition these two forces will be equal and opposite, i.e.
   $P \times \frac{\pi}{4} d^2 = \sigma \times \pi d$
   $$P = \frac{\sigma \times \pi d}{\frac{\pi}{4} d^2} = \frac{4\sigma}{d} \quad \text{…. (1.13)}$$
   From this equation we show that (with an increase in size of droplet the pressure intensity is decreases)

Notes :

- Property of exerting forces on fluids by fine tubes and porous media, due to both cohesion and adhesion
- Cohesion < adhesion, liquid wets solid, rises at point of contact
- Cohesion > adhesion, liquid surface depresses at point of contact, non-wetting fluid
- ***The contact angle*** is defined as the angle between the liquid and solid surface
- Meniscus: curved liquid surface that develops in a tube

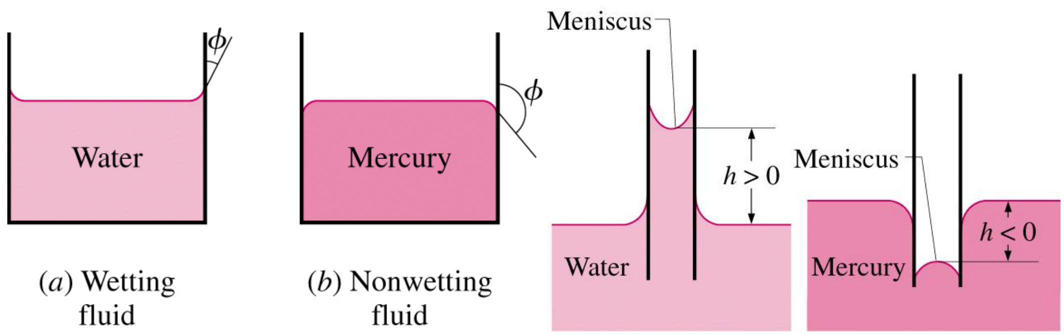


weight of fluid column = surface tension pulling force

$$\rho g(\pi R^2 h) = 2\pi R\sigma \cos\emptyset \quad .... (1.14)$$

$$h = \frac{2\sigma \cos\emptyset}{\rho g R} \quad .... (1.15)$$

● Expression above calculates the *approximate* capillary rise in a small tube
● The meniscus lifts a small amount of liquid near the tube walls, as *r* increases this amount may become insignificant
● Thus, the equation developed *overestimates* the amount of capillary rise or depression, particularly for large *r*.
● For a clean tube, $\emptyset = 0o$ for water, $\emptyset = 140o$ for mercury
● For $r > ¼$ *in* (*6 mm*), capillarity is negligible ● Its effects are negligible in most engineering situations.
● Important in problems involving capillary rise, e.g., soil water zone, water supply to plants
● When small tubes are used for measuring properties, e.g., pressure, account must be made for capillarity



Example (9) : ***If the surface tension of water-air interface is 0.069 N/m, what is the pressure inside the water droplet of diameter 0.009 mm?***

Solution:

Given d= 0.009 mm; σ= 0.069 N/m

The water droplet has only one surface, hence,

$$P = \frac{4\sigma}{d} = \frac{4\times0.069}{0.009\times10^{-3}} = 30667\,\frac{N}{m^2} = \mathbf{30.667\,\frac{kN}{m^2}\,or\,kPa}$$

Example (10) : ***A clean tube of diameter 2.5 mm is immersed in a liquid with a coefficient of surface tension = 0.4 N/m. the angle of contact of the liquid with the clean glass can be assumed to be 135º . the density of the liquid= 13600 kg/m³ . what would be the level of the liquid in tube relative to free surface of the liquid inside the tube?***

Solution:

Given d= 2.5 mm, σ= 4 N/m, $\emptyset = 135^{o}$; $\rho = 13600\ kg/m^3$

Level of the liquid in the tube, h:

$$\boldsymbol{h = \frac{2\sigma\,cos\emptyset}{\rho g R}}$$

$$h = \frac{4\times0.4\times cos135}{(9.81\times13600)\times2.5\times10^{-3}}$$

$$= -3.3910^{-3}m \quad or -3.39mm$$

Negative sign indicates that there is a capillary depression (fall) of 3.39 mm.

Home Work : ***Derive an expression for the capillary height change h, as shown, for a fluid of surface tension σ and contact angle Φ between two parallel plates W apart. Evaluate h for water at 20°C (σ=0.0728 N/m) if W = 0.5 mm.***

Solution:

With b the width of the plates into the paper, the capillary forces on each wall together balance the weight of water held above the reservoir free surface:

$$hb\rho gW = 2(\sigma b\cos\emptyset)$$

$$\mathrm{h} = \frac{2(\sigma\cos\emptyset)}{\rho gW}$$

for water at 20°C (σ=0.0728 N/m, $\gamma = 9790\ N/m^3$) and W = 0.5 mm.

$$\mathrm{h} = \frac{2 \times (0.0728 \times \cos(0))}{9790 \times 0.0005} = 0.03\mathrm{m} = 30\mathrm{mm}$$

Chapter Two

FLUID STATICS

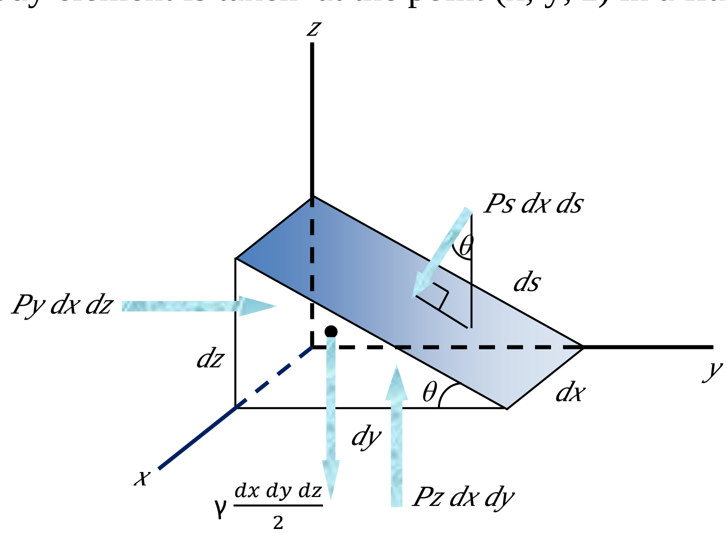
2.1. Introduction :

Fluid statics is that branch of mechanics of fluids that deals primarily with fluids at rest. Problems in fluid statics are much simpler than those associated with the motion of fluids, and exact analytical solutions are possible. Since individual elements of fluid do not move relative to one another, shear forces are not involved and all forces due to the pressure of the fluid are normal to the surfaces on which they act. Fluid statics may thus be extended to cover instances in which elements of the fluid do not move relative to one another even though the fluid as a whole may be moving. With no relative movement between the elements, the viscosity of the fluid is of no concern.

In this chapter we shall first examine the variation of pressure throughout an expanse of fluid. We shall then study the forces caused by pressure on solid surfaces in contact with the fluid, and also the effects (such as buoyancy) of these forces in certain circumstances.

2.2. Forces Exerted to an Element in a Fluid

At a point, fluid at rest has the same pressure in all direction and to prove this, a small wedge-shaped free body element is taken at the point (x, y, z) in a fluid at rest.



$$\sum f_x = P_x \cdot dydz - P_s \cdot dsdz \cdot sin\theta = 0 \quad .... \quad (2.1)$$

$$\sum f_y = P_y \cdot dxdz - P_s \cdot dsdz \cdot cos\theta - \frac{1}{2} dxdydz \cdot \gamma = 0 \quad .... \quad (2.2)$$

For unit width of element in z direction, and from the geometry of wedge we have the follows:

$$ds \cdot sin\theta = dy\,, \quad and \quad ds \cdot cos\theta = dx \; .... \quad (2.3)$$

Substitute of eq.3 in eqs. 1 and 2 and rearrange the terms yields:

$P_x = P_s$

$$P_y \cdot dx = P_s \cdot dx + \frac{1}{2} dydx \cdot \gamma \quad .... (2.4)$$

At a point the element limits to have an infinitesimal dimensions and then we can eliminate the term $(\frac{1}{2} dydx \cdot \gamma)$ from the above eq. because of it's a higher order of differential values. Thus we have at final that:

$$P_x = P_s = P_y \quad .... (2.5)$$

Where θ is an arbitrary angle, these results gives an important first principle of hydrostatics:

At a point, fluid at rest has the same pressure in all direction.

2.2.1. Pressure Variation:

- For static fluid, pressure varies only with elevation (depth) change within fluid.

To prove this real, we take a cubic fluid element as shown:

While fluid at rest, applying the equations of equilibrium on the element. That's yield:

1. ***In vertical direction y***

$$\sum f_y = P_y \cdot dxdz - \left(P_y + dP_y\right) \cdot dxdz - dxdydz \cdot \gamma = 0$$

$$P_y \cdot dxdz - P_y \cdot dxdz - dP_y \cdot dxdz - dxdydz \cdot \gamma = 0 \quad .... (2.6)$$

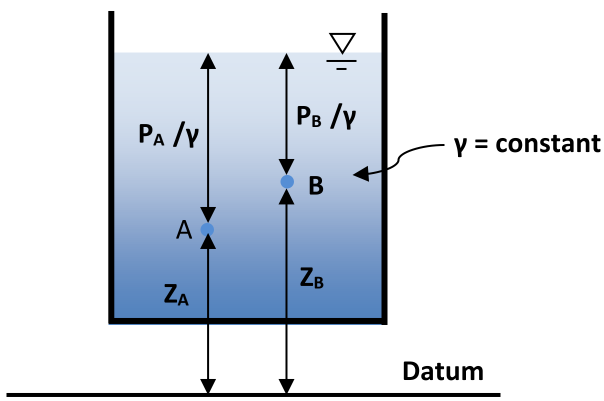
$$\rightarrow \quad dP_y \cdot dxdz = -dxdydz \cdot \gamma$$

$$\rightarrow \quad dP_y = -\gamma \cdot dy \quad .... (2.7)$$

When the direction of δy downward (m 17 n the negative direction of y), which is called depth and denoted by h, last above equation become:

$$dP_y = \gamma \cdot h \quad \ldots.(2.8)$$

these results gives an important second principle of hydrostatics:



- For static fluid, pressure varies only with elevation (depth) change within fluid by rate equal to specific weight γ of that fluid.
- In a fluid, pressure decreases linearly with increase in height
- $P_B - P_A = -\gamma\,(z_B - z_A)$
- This is the hydrostatic pressure change. With liquids we normally measure from the surface.
- Open free surface pressure in liquids is atmospheric, $P_{atmospheric}$.
- For constant density fluids, and taking the free surface pressure as zero, $p = \gamma h$.
- Thus $h = \frac{P}{\gamma}$
- Pressure related to the depth, h, of a fluid column.
- Referred to as the pressure head, *h*.

2. In horizontal direction x :

$$\sum f_x = P_x \cdot dydz - (P_x + dP_x) \cdot dydz = 0$$

$$P_x \cdot dydz - P_x \cdot dydz - dP_x \cdot dydz = 0 \quad \ldots .(2.9)$$

$$\rightarrow \; dP_x = 0 \quad \ldots .(2.10)$$

This equation means there is no change in horizontal pressure with horizontal direction.

Example 1 : ***A freshwater lake near Bristol, New Hampshire, has a maximum depth of 60m, and the mean atmospheric pressure is 91 kPa. Estimate the absolute pressure in kPa at this maximum depth.***

Solution:

From Table 2.1, take γ = 9790 N/m3. With P_a = 91 kPa and z = - 60 m, the pressure at this depth will be :

P = 91 kN/m^3 – (9790 N/m^3)(-60 m) $\frac{1\text{ kN}}{1000\text{ N}}$

P = 91 kP$_a$ + 587 kN/m^2 = 678 kP$_a$

By omitting P$_a$ we could state the result as P = 587 kP$_a$ (gage).

Example 2: ***The liquid oxygen(LOX) tank of a Saturn moon rocket is partially filled to a depth of 30 ft with LOX at - 320 ºF . The Pressure in the vapor above the liquid surface is maintained at 14.7 psia. Calculate the pressure at the inlet valve at the bottom of the tank.***

Solution:

γ = ρ * g = 2.33 *32.2 = 75 lb /ft^3

P$_1$ = 75 * 30 + 14.7 * 144 = 2341 psfa = 16.3 psia

Calculate pressure and specific weight at air in the U.S. standard atmosphere at altitude 35,000 ft.

Solution:

At sea level :

$P_1 = 14.7$ psia , $T_1 = 519^o$ R , $\gamma_1 = 0.0765$ lb/ft^3

$\frac{P}{\gamma^{1.235}} = \frac{14.7*144}{(0.0765)^{1.235}}$, $\quad \frac{1}{\gamma} = \frac{6450}{P^{0.81}}$

$35{,}000 = 6450 \int_{p_2}^{14.7*144} P^{-0.81}\, dp$, $P_2 = 504$ psf = 3.5 psia

$T_2 = 519 - 35{,}000 * 0.00356 = 394^o$ R

$\gamma_2 = 504 * 32.2 / 1715 * 394 = 0.024$ lb/ft^3

2.2.2. Absolute and Gauge Pressure :

In practice, pressure is always measured by the determination of a pressure difference. If the difference is that between the pressure of the fluid in question and that of a vacuum then the result is known as the *absolute pressure* of the fluid. More usually, however, the difference determined is that between the pressure of the fluid concerned and the pressure of the surrounding atmosphere.

This is the difference normally recorded by pressure gauges and so is known as *gauge pressure*.

●If $P < P_{atm}$, we call it a vacuum (or negative or suction) pressure , its gage value = how much below atmospheric

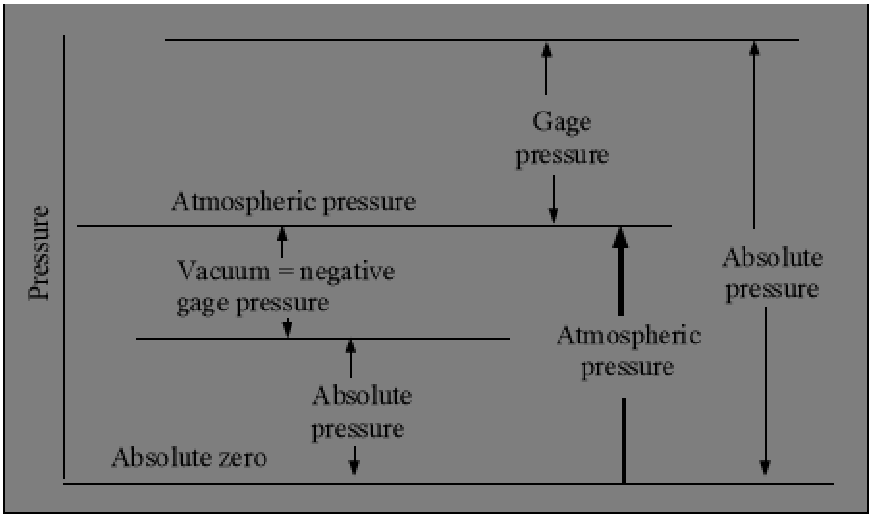
●Absolute pressure values are all positive

●While gage pressures may be either :

– Positive: if above atmospheric, or

– Negative (vacuum, suction): if below atmospheric

●Relationship between absolute, gage and atmospheric pressure reading: $P_{abs} = P_{atm} + P_{gage}$



Pressure Units

	Pascal (Pa)	bar (bar)	technical atmosphere (at)	atmosphere (atm)	torr (Torr)	pound-force /in^2 (psi)
1 Pa	≡1 N/m^2	10^{-5}	1.0197×10^{-5}	9.8692×10^{-6}	7.5006×10^{-3}	145.04×10^{-6}
1 bar	100,000	≡10^6 dyn/cm^2	1.0197	0.98692	750.06	14.5037744
1 at	98,066.5	0.980665	≡1 kgf/cm^2	0.96784	735.56	14.223
1 atm	101,325	1.01325	1.0332	≡1 **atm**	760	14.696
1 torr	133.322	1.3332×10^{-3}	1.3595×10^{-3}	1.3158×10^{-3}	≡1 Torr; ≈ 1 mmHg	19.337×10^{-3}
1 psi	6.894×10^3	68.948×10^{-3}	70.307×10^{-3}	68.046×10^{-3}	51.715	≡1 lbf/in^2

Example 3 : ***1 Pa = 1 N/m2 = 10−5 bar = 10.197×10−6 at = 9.8692×10−6 atm , etc.***

A pressure of 1 atm can also be stated as:

≡ 1.013 25 bar

≡ 1013.25 hectopascal (hPa)

≡ 1013.25 millibars (mbar, also mb)

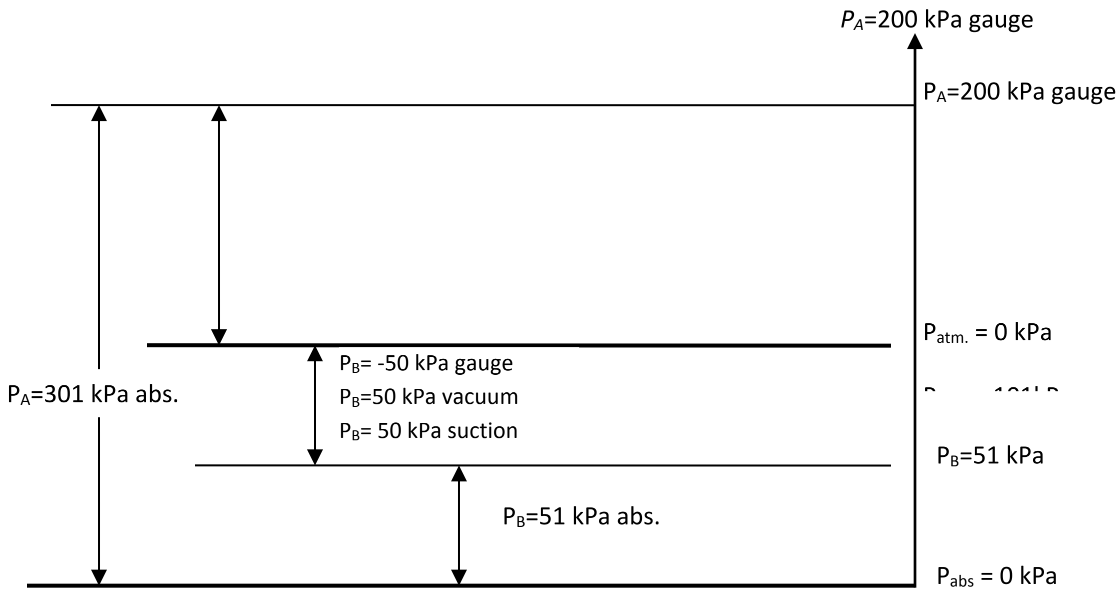
≡ 760 torr [B]

≈ 760.001 mm-Hg, 0 °C≈ 1.033 227 452 799 886 kgf/cm²

≈ 1.033 227 452 799 886 technical atmosphere

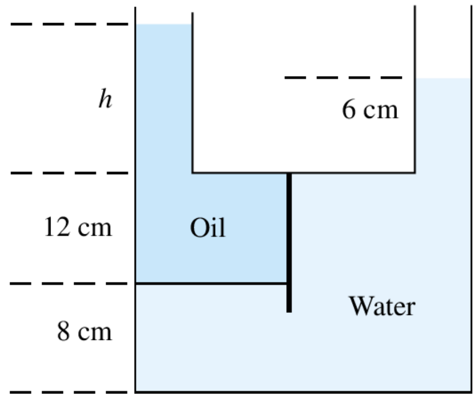
≈ 1033.227 452 799 886 cm–H_2O, 4 °C

Example 4 : ***For two points A, B: PA = 200 kPa gauge , PB = 51 kPa abs. , where P_{atm} = 101 kPa , Re-read the value of pressures in A and B in all possible forms.***



Example 5 : ***In Fig. the tank contains water and immiscible oil at 20°C. What is h in cm if the density of the oil is 898 kg/m3?***

Solution: For water take the density = 998 kg/m^3. Apply the hydrostatic relation from the oil surface to the water surface, skipping the 8-cm part:

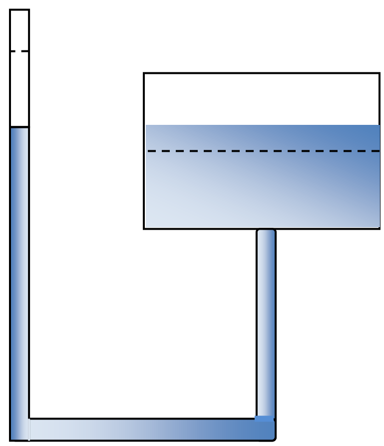


$$p_{atm} + (898)(g)(h + 0.12) - (998)(g)(0.06 + 0.12) = p_{atm},$$

Solve for $h \approx 0.08 \text{ m} \approx \mathbf{8.0\ cm}$ *Ans.*

Home Works :

1. A closed circular tank filled with water and connected by a U-piezometric tube as shown in figure. At the beginning the pressure above the water table in the tank is atmospheric, then the gauge that connected with tank read an increasing in pressure that caused falling in the water level in the tank by 3 cm. a) calculate the deference in height that accrued between water levels inside the tank and in the external tube leg. b) Determine the final pressure that was reading by the gauge.



2. One means of determining the surface level of oil in a tank is by discharging a small amount of air through a small tube, the end of which is submerged in the tank, and reading the pressure on the gauge that is tapped into the tube. Then the level of the oil surface in the tank can be calculated. If the pressure on the gauge is 30 kPa and the specific gravity of oil is $S = 0.85$, what is the depth d of oil in the tank?

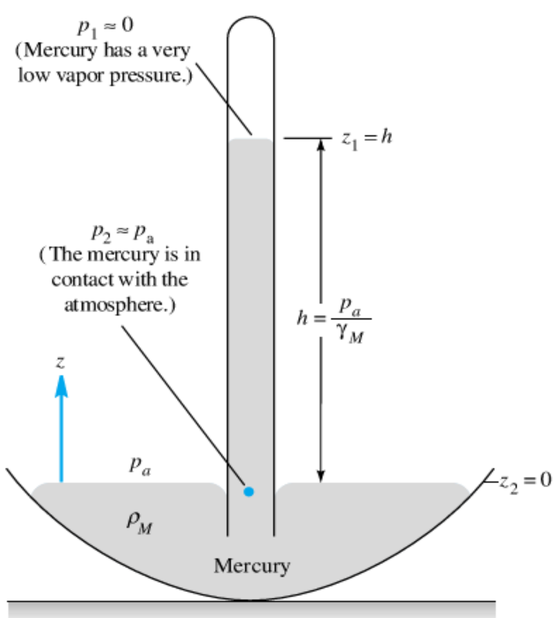
2.3. Pressure measurement devices :

2.3.1. Absolute pressure measurement (Barometers) :

Barometers : The instrument used to measure atmospheric pressure is called barometer

1. **Mercury Barometer**: which is illustrated in figure below, which consist of a one meter length tube filled with mercury and inverted into a pan that's filled partially with mercury. The height difference of mercury in inverted tube respect to outside them reads the atmospheric pressure value.

- Values of standard sea-level atmospheric pressure=*101.325 kPa abs =1013.25 mbar abs*
  = 760 mm Hg
  =10.34 m H_2O

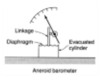


(a)

(b)

A barometer measures local absolute atmospheric pressure: (*a*) the height of a mercury column is proportional to p_{atm}; (*b*) a modern portable barometer, with digital readout, uses the resonating silicon element

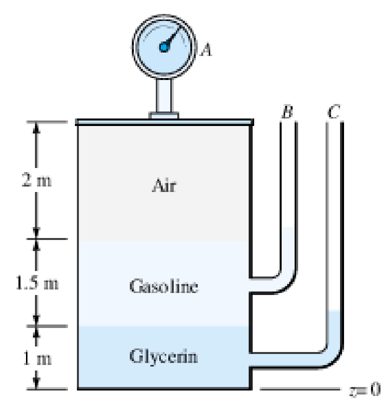
2. *Aneroid barometer:* uses elastic diaphragm to measure atmospheric pressure



***Example 6:* (Piezometer)**

In figure pressure gage A reads 1.5 kPa. The fluids are at 20oC. Determine the elevations z, in meters, of the liquid levels in the open piezometer tubes B and C.

Solution : (B) Let piezometer tube B be an arbitrary distance H above the gasoline- glycerin interface. The specific weight are γ_{air} = 12 N/m^3 , $\gamma_{gasoline}$ = 6670 N/m^3 , and $\gamma_{glycerin}$ = 12360 N/m^3 . then apply the hydrostatic formula from point A to point B.



$$1500\ \text{N/m}^2 + (12.0\ \text{N/m}^3)(2.0\ \text{m}) + 6670(1.5 - \text{H}) - 6670(Z_B - \text{H} - 1.0) = p_B = 0\ (\text{gage})$$

Solve for $Z_B = \mathbf{2.73\ m}$ (23 cm above the gasoline-air interface) *Ans.* (b)

Solution (C): Let piezometer tube C be an arbitrary distance Y above the bottom. Then

$$1500 + 12.0(2.0) + 6670(1.5) + 12360(1.0 - \text{Y}) - 12360(Z_C - \text{Y}) = p_C = 0\ (\text{gage})$$

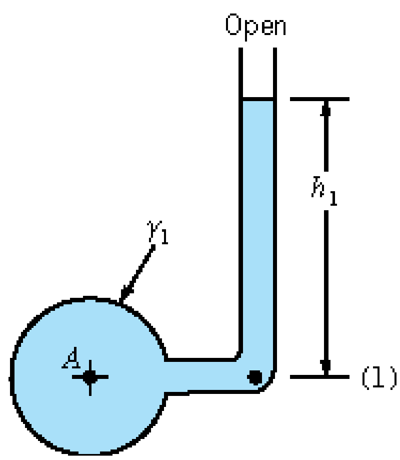
Solve for $Z_C = \mathbf{1.93\ m}$ (93 cm above the gasoline-glycerin interface) *Ans.* (c)

2.3.2.Gage pressure measurement:

2.3.2.1. Manometry

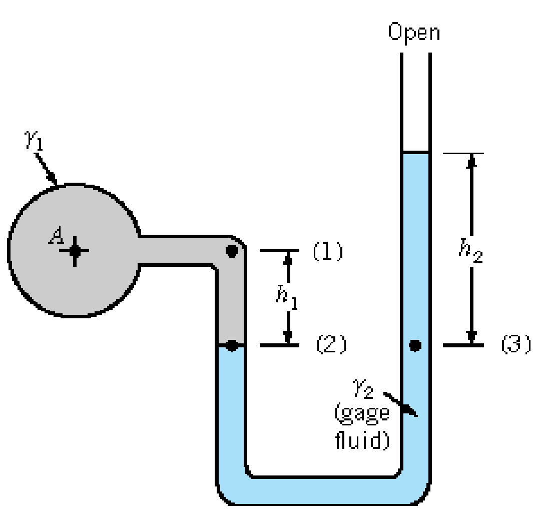
1. **Piezometer**

For measuring pressure inside a vessel or pipe in which liquid is there, a tube open at the top to atmosphere may be attached, tapped, to the walls of the container (or pipe or vessel) containing liquid at a pressure (higher than atmospheric) to be measured, so liquid can rise in the tube. By determining the height to which liquid rises and using the relation P1 = ρgh, gauge pressure of the liquid can be determined. Such a device is known as piezometer. To avoid capillary effects, a piezometer's tube should be about 12mm or greater.



2. **Manometers**

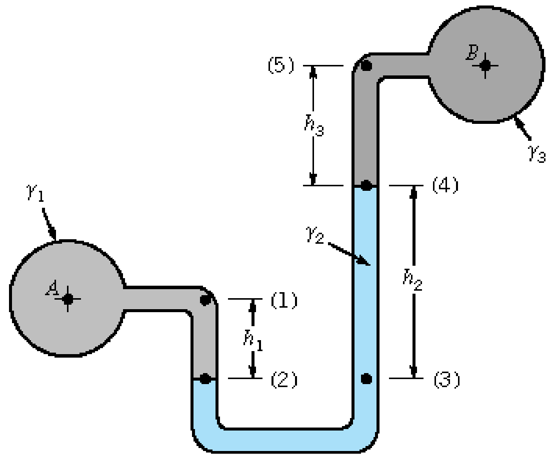
For high pressures measurement using piezmeters become impractical and it is useless for pressure measurement in gases. The manometers in its various forms is an extremely useful type of pressure measuring instrument for these cases.



Professor John Foss (Michigan State University) Procedure for manometers pressure calculation:

$$\boldsymbol{P_{down} = P^{up} + \gamma h}$$

$$\boldsymbol{P^{up} = P_{down} - \gamma h}$$

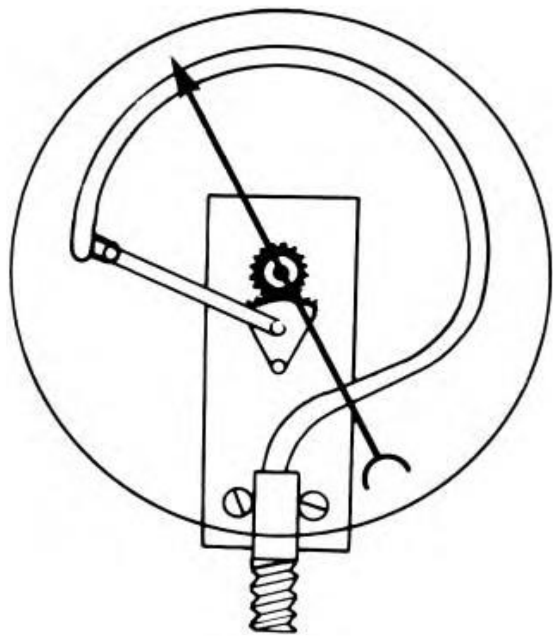


. *Manometers limitations:* manometers suffers from a number of limitations.

1. While it can be adapted to measure very small pressure differences, it cannot be used conveniently for large pressure differences - although it is possible to connect a number of manometers in series and to use mercury as the manometric fluid to improve the range. (limitation)
2. A manometer does not have to be calibrated against any standard; the pressure difference can be calculated from second and third principles in hydrostatics. (Advantage)
3. Some liquids are unsuitable for use because they do not form well-defined interface. Surface tension can also cause errors due to capillary rise; this can be avoided if the diameters of the tubes are sufficiently large - preferably not less than 12 mm diameter. (limitation)
4. A major disadvantage of the manometer is its slow response, which makes it unsuitable for measuring fluctuating pressures
5. It is essential that the pipes connecting the manometer to the pipe or vessel containing the liquid under pressure should be filled with this liquid and there should be no air bubbles in the liquid.(limitation).

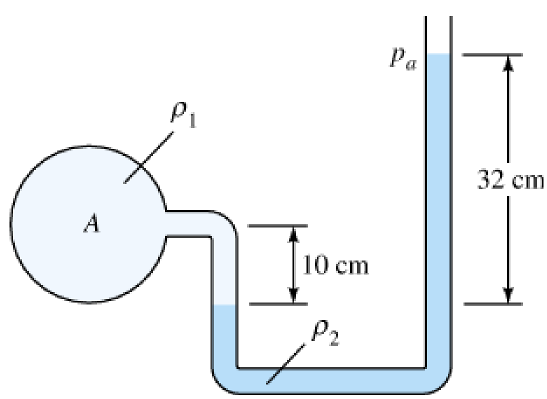
2.3.2.2. Bourdon gage

Curved tube of elliptical cross-section changes curvature with changes in pressure. Moving end of tube rotates a hand on a dial through a linkage system. Pressure indicated by gage graduated in kPa or kg/cm^2 (=98.0665 kPa) or psi or other pressure units.

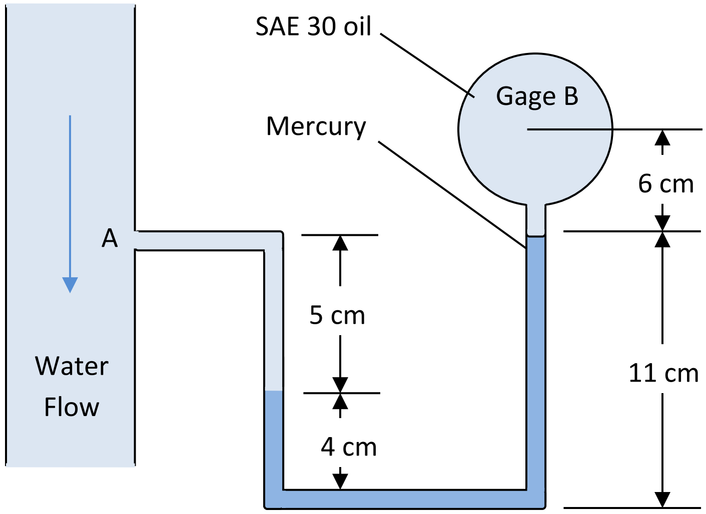


Example 7: (U-manometers)

In Figure fluid 1 is oil (Sg=0.87) and fluid 2 is glycerin at 20oC (γ=12360 N/m3). If Pa=98 kPa, determine the absolute pressure at point A



Example 8: ***(Differential-Manometers)Pressure gage B is to measure at point A in a water flow. If the pressure at B is 87 Kpa, estimate the pressure at A, in Kpa. Assume all fluids are at 20° C.***



Solution

First list the specific weights from Table 2.1 or Table A.3:

$$\gamma_{\text{water}} = 9790 \text{ N/m}^3 \qquad \gamma_{\text{mercury}} = 133{,}100 \text{ N/m}^3 \qquad \gamma_{\text{oil}} = 8720 \text{ N/m}^3$$

Now proceed from A to B, calculating the pressure change in each fluid and adding:

$$p_A - \gamma_W(\Delta z)_W - \gamma_M(\Delta z)_M - \gamma_O(\Delta z)_O = p_B$$

or $$p_A - (9790 \text{ N/m}^3)(-0.05 \text{ m}) - (133{,}100 \text{ N/m}^3)(0.07 \text{ m}) - (8720 \text{ N/m}^3)(0.06 \text{ m})$$

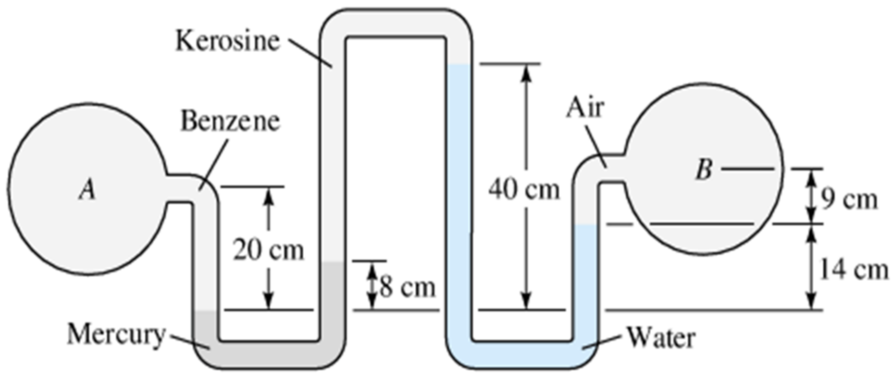
$$= p_A + 489.5 \text{ Pa} - 9317 \text{ Pa} - 523.2 \text{ Pa} = p_B = 87{,}000 \text{ Pa}$$

where we replace N/m^2 by its short name, Pa. The value $\Delta z_M = 0.07$ m is the net elevation change in the mercury (11 cm − 4 cm). Solving for the pressure at point A, we obtain

$$p_A = 96{,}351 \text{ Pa} = 96.4 \text{ kPa} \qquad \textit{Ans.}$$

Example 9: ***(Differential-Manometers)***

In the figure below, all fluids are at 20ºC. Determine the pressure difference (P_a) between points A and B.



Solution: Take the specific weights to be

Benzene: 8640 N/m^3 Mercury: 133100 N/m^3

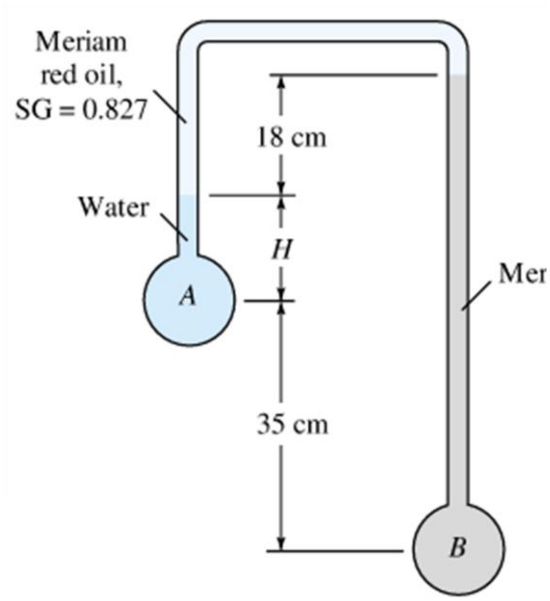
Kerosene: 7885 N/m^3 Water: 9790 N/m^3

and γ_{air} will be small, probably around 12 N/m^3. Work your way around from A to B:

$$p_A + (8640)(0.20\text{ m}) - (133100)(0.08) - (7885)(0.32) + (9790)(0.26) - (12)(0.09)$$
$$= p_B, \quad \text{or, after cleaning up,} \quad p_A - p_B \approx \mathbf{8900\ Pa} \quad \textit{Ans.}$$

Example 10: **(Inverted-Manometers)**

For the inverted manometer of the figure below, all fluids are at 20ºC. If $P_B - P_A$ = 97 kPa, what must the height H be in cm?



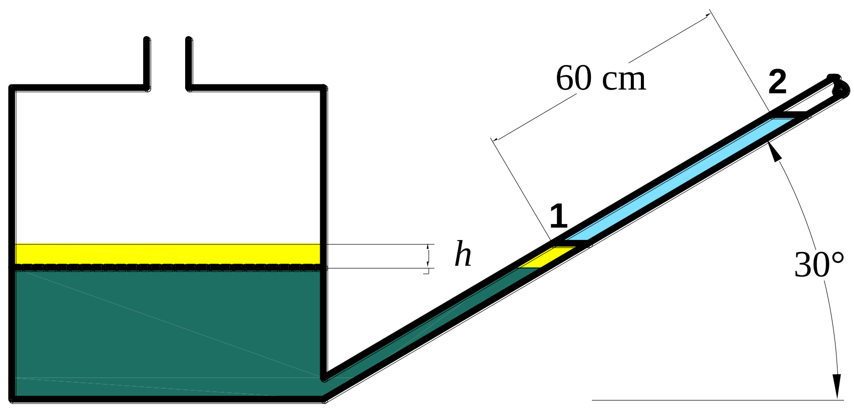
Solution: Gamma = 9790 N/m^3 for water and 133100 N/m^3 for mercury and (0.827)(9790) = 8096 N/m^3 for Meriam red oil. Work your way around from point A to point B:

$$p_A - (9790\ N/m^3)(H\ meters) - 8096(0.18)$$
$$+133100(0.18 + H + 0.35) = p_B = p_A + 97000.$$

Solve for $H \approx 0.226\ m = \mathbf{22.6\ cm}$ *Ans.*

Example 11: ***(inclined-Manometers (micro-manometer)***

The ratio of the cistern diameter to tube diameter is 10, when air in the cistern is atmospheric pressure, the free surface in the tube is at position 1. When the cistern is pressurized, the liquid in the tube moves 60 cm up the tube from position 1 to position 2. What the cistern pressure that cause this deflection? The specific gravity of the liquid is 1.2.



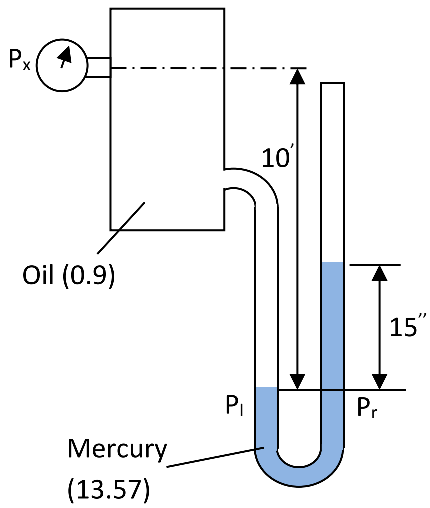
Solution:

$P_2 = 0.0$

$$P_1 = P_2 + 1.2 \times 9810 \times 0.6 \times \sin(30) = 11772 \times 0.3 = 3531.6\ Pa$$
$$P_{cistern} = P_1 + h \times 1.2 \times 9810$$
$$D_{cistern} = 10\ d_{tube}$$
$$\Delta V_{cistern} = \Delta V_{tube}$$
$$\frac{D^2}{4} \times \pi \times h = \frac{d^2}{4} \times \pi \times 0.6$$
$$\frac{100 \times d^2}{4} \times \pi \times h = \frac{d^2}{4} \times \pi \times 0.6$$
$$h = .006 = 0.6\ cm$$
$$P_{cistern} = 3531.6 + 0.006 \times 1.2 \times 9810 = 3602.2\ Pa$$

***Example 12:* (U – Manometer)**

This vertical pipe line with attached gage and manometer contains oil and mercury as shown. The manometer is open to the atmosphere. There is no flow in the pipe. What will be the gage reading P_x ?



Solution :

$P_l = P_x + (0.9 * 62.4) * 10$

$P_l = (13.57 * 62.4) * (15/12)$

Because

$P_l = P_r$

$P_x = 497$ psf $= 3.45$ psi

Chapter Three

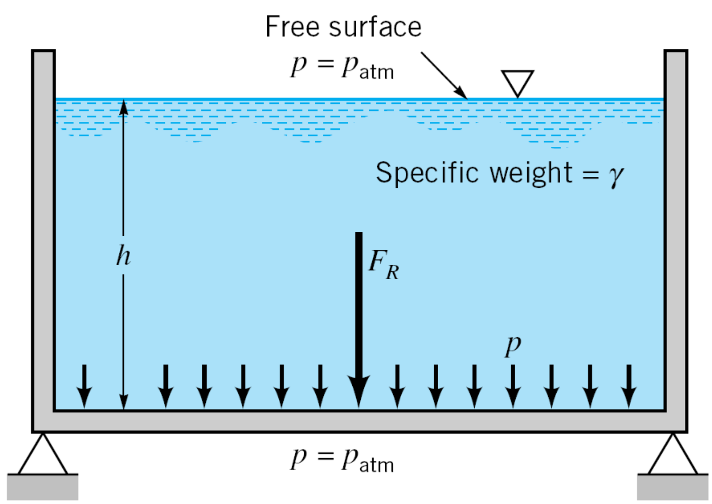
HYDROSTATIC FORCES

3.1. Introduction :

The pressure of a fluid causes a thrust to be exerted on every part of any surface with which the fluid is in contact. The individual forces distributed over the area have a resultant, and determination of the magnitude, direction and position of this resultant force is frequently important. For a plane *horizontal* surface at which the fluid is in equilibrium the matter is simple: the pressure does not vary over the plane and the total force is given by the product of the pressure and the area. Its direction is perpendicular to the plane – downwards on the upper face, upwards on the lower face – and its position is at the centroid of the plane. But if the surface is not horizontal the pressure varies from one point of the surface to another and the calculation of the total thrust is a little less simple.

3.2. Hydrostatic Forces on Plane Surfaces :

3.2.2.1. On Horizontal Surface:

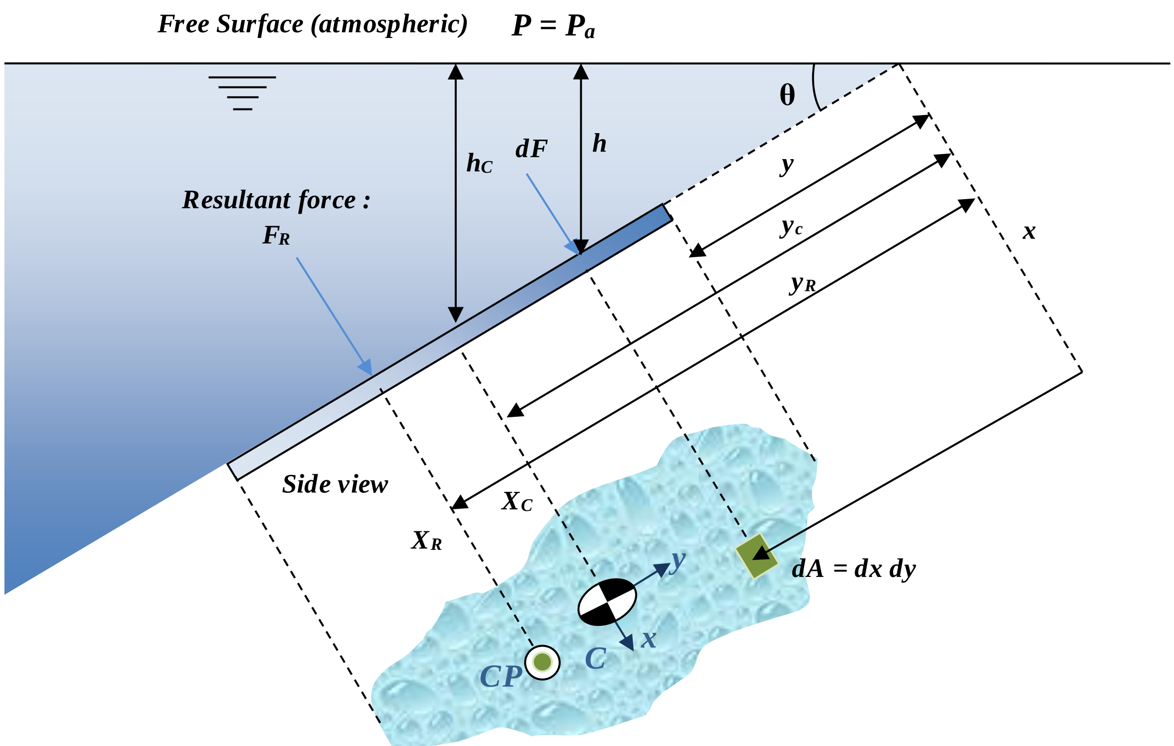


∴ Resultant force is :

$F_P = PA = \gamma h\ A$ …. (3.1)

A: the bottom area of container

3.2.2.2. On an Inclined Surface:



Hydrostatic force and center of pressure on an arbitrary plane surface of area A inclined at an angle θ below the free surface.

Consider a plane shown above :

- At surface: $p = p_{atm}$

- Angle θ between free surface & the inclined plane

- Along the vertical depth h

  ☞ Pressure linearly changes ☞ Hydrostatic force changes

Differential Force acting on the differential area dA of plane, (Perpendicular to plane):

$dF = (\text{pressure}).(\text{Area}) = (\gamma h).(dA)$ …. (3.2)

Then, Magnitude of ***total*** resultant force F_P

$F_P = \int_A \gamma\,(\sin\theta)\,dA \;\; \dots.(3.3)$ where $h = y\sin\theta$

Or : $F_P = \gamma\sin\theta \int_A y\,dA \;\; \dots.(3.4)$ (1st m of the area about x-axis)

$$y_c = \frac{\int_A y\,dA}{A} \;\; \dots.(3.5)$$

$$x_c = \frac{\int_A x\,dA}{A} \;\; \dots.(3.6)$$

y_c , x_c : *y* and *x* coordinates of the center of area respectively (Centroid)

$$MX_C = \int_M xdm \;\; \dots.(3.7)$$

$$MY_C = \int_M ydm \;\; \dots.(3.8)$$

Y_C , X_C : *y* and *x* coordinates of the center of mass respectively (first moment)

$I = \int_M r^2 dm \;\; \dots.(3.9)$ (second moment of mass)

$$I_x = \int_A y^2 dA \;\; \dots.(3.10)$$

$$I_y = \int_A x^2 dA \;\; \dots.(3.11)$$

I_x , I_y : second moment (moment of inertia) about x and y coordinates respectively, then:

$F_P = \gamma\, A\, y_C \sin\theta = (\gamma\, h_C)\, A \;\; \dots.(3.12)$

where $\gamma\, h_c$: Pressure at the centroid

$\gamma\, h_c$ = (Pressure at the centroid) × Area

The location of point of action of F_R

- ***Not passing though Centroid*** & related with the ***balance of torques*** due to of F_P

i) Position of F_P on *y*-axis

y_P : *y* coordinate of the point of action of F_P

Taking moment about x axis:

The moment of resultant force = The moment of its components

$$F_P y_P = \int_A y dF$$

$$\Longrightarrow (\cancel{\gamma} A y_c \cancel{\sin}\theta) y_R = \int_A \gamma \cancel{\sin}\theta y^2 dA = \gamma \cancel{\sin}\theta \int_A y^2 dA$$

$$y_P = \frac{\int_A y^2 dA}{y_c A} = \frac{I_x}{y_c A} \quad \ldots .(3.13)$$

where $I_x = \int_A y^2 dA$: *2nd moment of area (*Moment of inertia, +ve always *)*

or, by using the parallel-axis theorem, $I_x = I_{xc} + A{y_c}^2$

$$y_P = \frac{I_{xc}}{y_c A} + y_c \quad \ldots .(3.14)$$ (***Always below the centroid***)

ii) Position of FP on x-axis

x_P : x coordinate of the point of action of F_P

The moment of resultant force = The moment of its components

$$F_P x_P = \int_A x dF$$

$$\Longrightarrow (\cancel{\gamma} A y_c \cancel{\sin}\theta) x_P = \int_A \gamma \cancel{\sin}\theta x y dA = \gamma \cancel{\sin}\theta \int_A x y dA$$

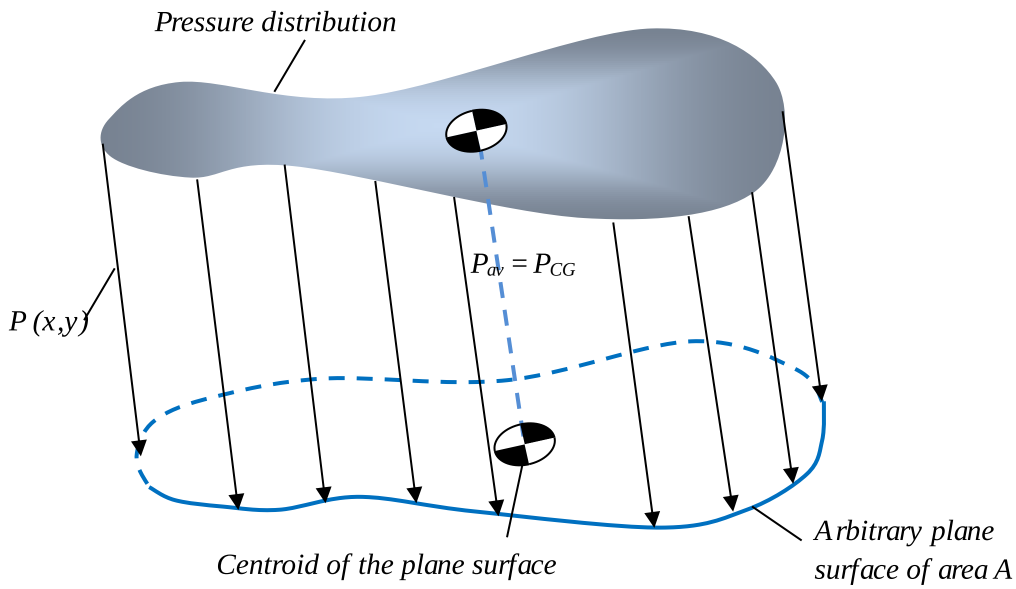
$$x_P = \frac{\int_A xydA}{y_c A} = \frac{I_{xy}}{y_c A} \quad \ldots .(3.15)$$

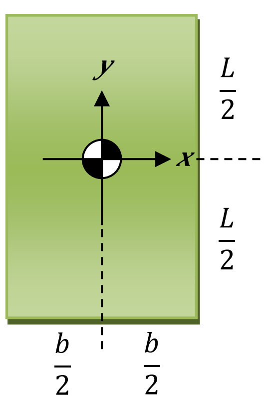
where $\int_A xydA = I_{xy}$: **Area product of inertia (+ve or –ve)**

or, by using the perpendicular-axis theorem, $I_{xy} = I_{xyc} + A x_c y_c$

$$x_P = \frac{I_{xyc}}{y_c A} + x_c \quad \ldots .(3.16)$$

c.f. For a ***symmetric*** submerged area, $x_P = x_c$ ($I_{xyc} = 0$)

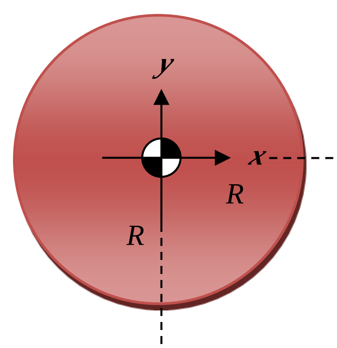




$A = bL$

$I_{xx} = \dfrac{bh^3}{12}$

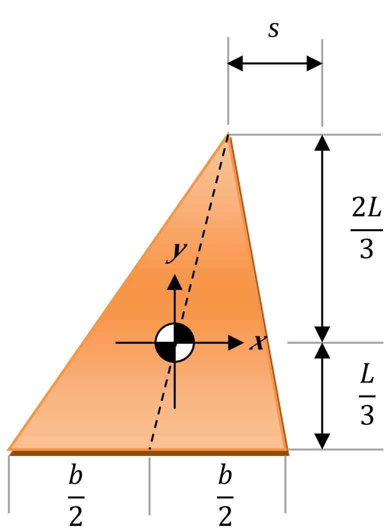
$I_{xy} = 0$



$A = \pi R^2$

$I_{xx} = \dfrac{\pi R^4}{4}$

$I_{xy} = 0$



$A = bL/2$

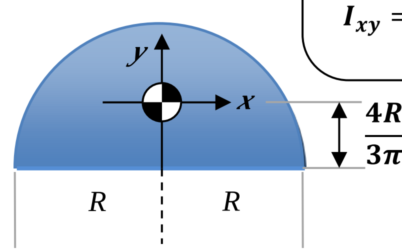
$I_{xx} = \dfrac{bh^3}{36}$

$I_{xy} = \dfrac{b(b-2s)L^2}{72}$

$A = \dfrac{\pi R^2}{2}$

$I_{xx} = 0.10976\, R^4$

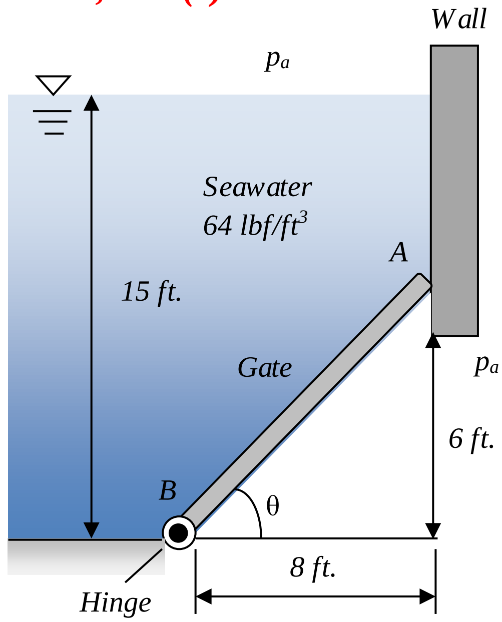
$I_{xy} = 0$



Moments of Inertia of Several Common Cross Sections

Example 1 : ***The gate in Fig. below is 5 ft wide, is hinged at point B, and rests against a smooth wall at point A. Compute (a) the force on the gate due to seawater pressure, (b) the horizontal force P exerted by the wall at point A, and (c) the reactions at the hinge B.***

Solution :



Part a :

By geometry the gate is 10 ft long from *A* to *B*, and its centroid is halfway between, or at elevation 3 ft above point *B*. The depth h_{CG} is thus 15- 3 = 12 ft. The gate area is 5(10) = 50 ft^2. Neglect p_a as acting on both sides of the gate. From Eq. (3.4) the hydrostatic force on the gate is :

$F = p_{CG}A = \gamma h_{CG}A = (64\ \text{lbf/ft}^3)(12\ \text{ft})(50\ \text{ft}^2) = 38{,}400\ \text{lbf}$ **Ans. (a)**

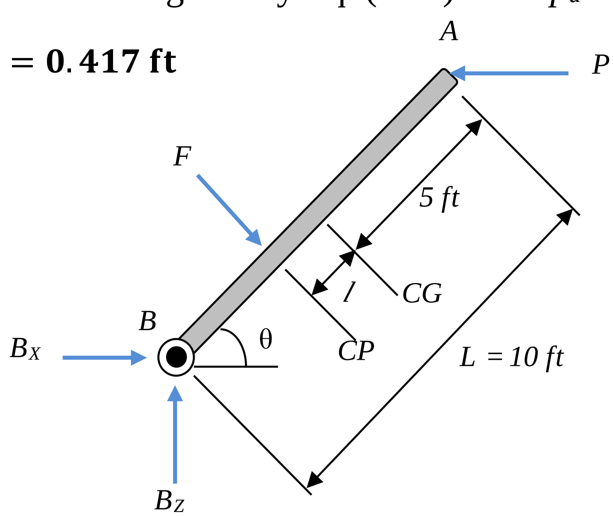
Part b :

First we must find the center of pressure of *F*. A free-body diagram of the gate is shown in figure below. The gate is a rectangle, hence

$I_{xy} = 0$ and $I_{xx} = bL^3/12 = 5*10^3/12 = 417\ \text{ft}^4$

The distance *l* from the CG to the CP is given by Eq. (3.10) since p_a is neglected.

$$l = -y_{CP} = +\frac{I_{xx}\sin\theta}{h_{CG}A} = \frac{417\left(\frac{6}{10}\right)}{12*50} = \mathbf{0.417\ ft}$$



The distance from point *B* to force *F* is thus 10-*l*-5 = 4.583 ft. Summing moments counterclockwise about *B* gives :

$PL \sin\theta - F(5-l) = P*6 - 38400 * 4.583 = 0$

$P = 29300$ ft.

Ans. (b)

Part c :

With *F* and *P* known, the reactions *Bx* and *Bz* are found by summing forces on the gate

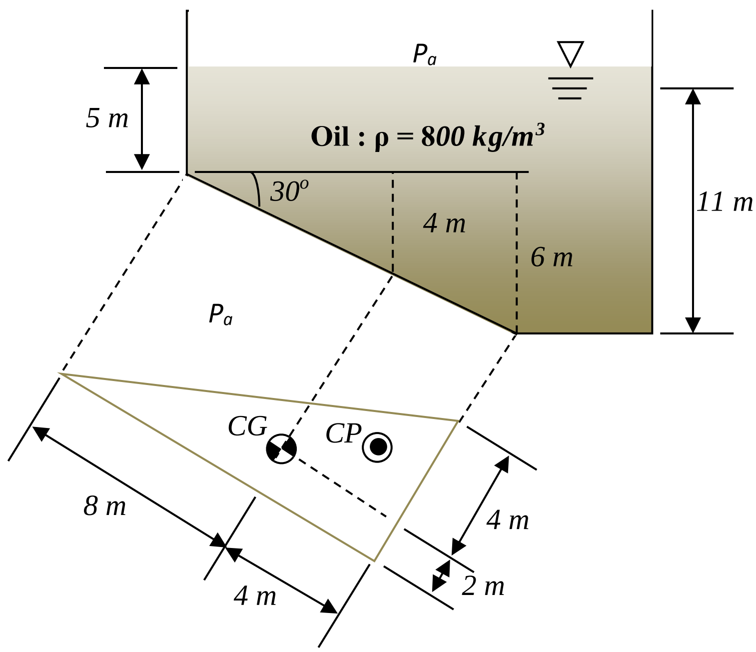
$\Sigma F_x = 0 = B_x + F \sin\theta - P = B_x + 38400 * 0.6 - 29300$

$B_x = 6300$ lbf

$\Sigma F_z = 0 = B_z - F \cos\theta = B_z - 38400 * 0.8$

$B_z = 30700$ lbf **Ans. (c)**

Example 2: ***A tank of oil has a right-triangular panel near the bottom, as in figure below Omitting p_a, find the (a) hydrostatic force and (b) CP on the panel.***



Solution :

Part a :

The triangle has properties given above. The centroid is one-third up (4 m) and one-third over (2 m) from the lower left corner, as shown. The area is

$A = 0.5*6*12 = 36\ m^2$

The moment of inertia are :

$I_{xx} = bL^3/36 = 6*12^3/36 = 288\ m^4$

$$I_{xy} = \frac{b(b-2s)L^2}{72} = \frac{6[6-2*6]*12^2}{72} = -72\ m^4$$

The depth to the centroid is $h_{CG} = 5 + 4 = 9$ m; thus the hydrostatic force from Eq. (3.10) is
$F = \rho\ g\ h_{CG}\ A = 800*\ 9.807 * 9 * 36 = 2.54*10^6\ (kg.m)/s^2 = 2.54*10^6\ N = 2.54\ MN$ **Ans. (a)**

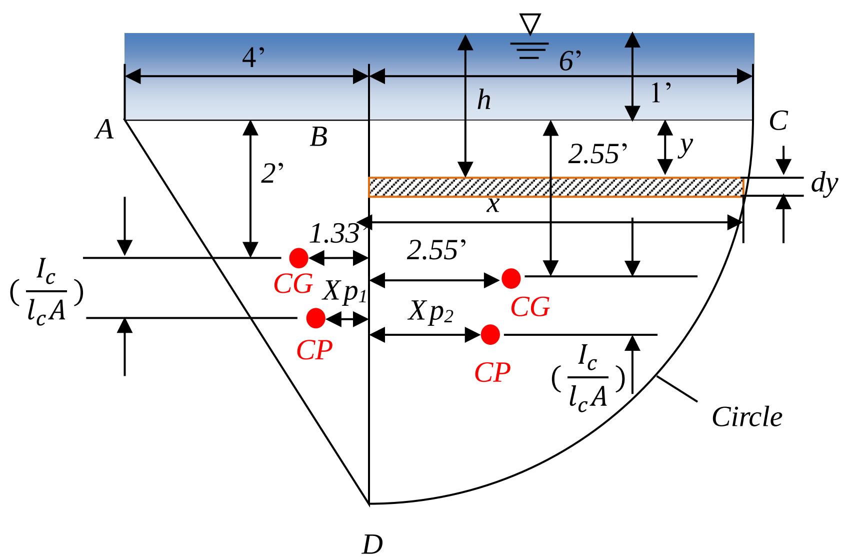
Part (b):

$$\mathbf{y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A} = -\frac{288\sin 30^o}{9*36} = -0.444\ m}$$

$$\mathbf{x_{CP} = -\frac{I_{xy}\sin\theta}{h_{CG}A} = -\frac{(-72)\sin 30^o}{9*36} = +0.111\ m}$$ **Ans. (b)**

The resultant force F = 2.54 MN acts through this point, which is down and to the right of the centroid, as shown in figure above.

***Example 3:* Calculate magnitude, direction and location of the total force exerted by the water on one side of this composite area which lies in a vertical plane.**



Solution:

By inspection the direction of the force is normal to the area.

Magnitude :

Force on triangle = 62.4*3*12 = 2245 lb

Force on quadrant = 62.4*3.55*(36π/4) = 6275 lb

Total force of composite area = 2245 + 6275 = 8520 lb

Vertical location of resultant force :

$\frac{I_c}{l_c A}$ for triangle $= \frac{4*6^3}{36*3*12} = 0.667$ ft

$\frac{I_c}{l_c A}$ for quadrant $= \frac{71.1}{3.55*28.3} = 0.71$ ft ,taking moment about line AC,

2245*2.667 + 6275*3.26 = 8520 (l_p – 1)

l_p = 4.09 ft

lateral location of resultant force : since the center of pressure of the triangle is on the median line , x_{p1} , is given (from similar triangles) by

$\frac{(1.333 - x_{p1}}{2} = \frac{0.667}{6}$

x_{p1} = 1.111 ft

Dividing the quadrant into horizontal strips of differential height, dy , the moment about BD of the force on any one of them is :

dM = (x dy) γ h ($\frac{x}{2}$)

in which h = y + 1 and $x^2 + y^2 = 36$, substituting and integrating gives the moment about BD of the total force on the quadrant,

$\mathrm{M} = \frac{62.4}{2}\int_0^6 (36 - \mathrm{y}^2)(\mathrm{y}+1)\mathrm{dy} = 14600 \text{ ft.lb}$

and thus

x_{p2} = 14600/6275 = 2.33 ft

finally taking moment about line BD,

2.33 * 6275 – 2245*1.111 = 8520 * x_p

x_p = 1.425 ft

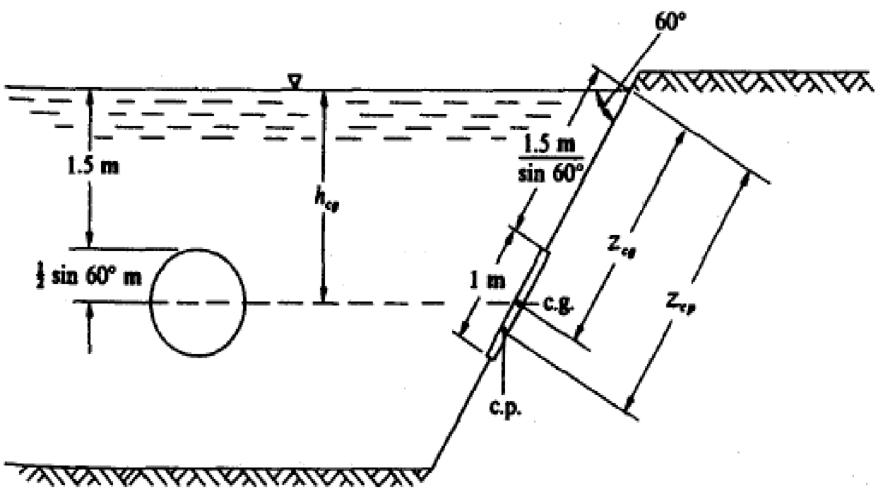
thus the center of pressure of the composite figure is 1.425 ft to the right of BD and 4.1 ft below the water surface.

Example 4: ***An inclined, circular gate with water on one side as shown in figure. Determine the total resultant force acting on the gate and the location of the centre of pressure.***

Solution:

$F = \gamma\, h_{cg}\, A = (9.79)[1.5+0.5(\pi(1)^2/4] = 14.86$ kN

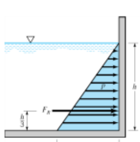
$$Z_{cp} = Z_{cg} + \frac{I_{cg}}{Z_{cg}A} = \left[\frac{1.5}{\sin 60} + 0.5 * 1\right] + \frac{\pi\frac{(1)^4}{64}}{[1.5/\sin 60 + 0.5*(1)][\pi*\frac{(1)^2}{4}]} = 2.26\ m$$

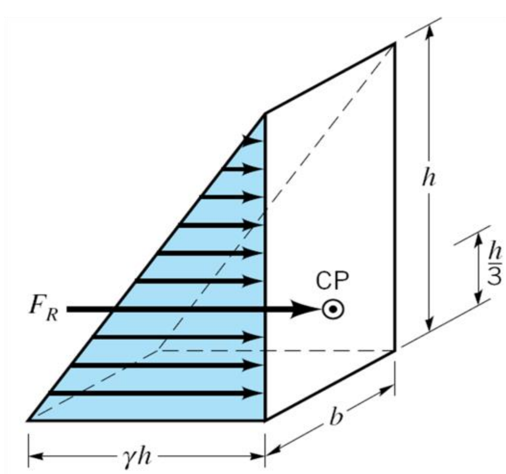


3.3. *Hydrostatic Forces of Pressure Prism* :

Especially applied for a rectangular surfaces (areas) **,** a Simple method for ***finding the force*** and ***the point of action*.**

Consider the situation shown :





Information from the diagram

- Vertical wall of width *b* and height *h*
- Contained liquid with specific weight γ
- Pressure: $p_{top} = 0$ & $p_{bottom} = \gamma\, h$

From the last section,

$$F_P = (\gamma h_c)\cdot(A) = p_{av} \text{ (at the centroid)} \times \text{area} = \gamma\left(\frac{h}{2}\right)A$$

Let's define a pressure-area space. (See the right figure above]

1. Horizontal axis: Magnitude of the pressure
2. Vertical axis: Height of the area
3. Axis toward the plane: Width of the area

: Resultant volume (***Pressure prism***)

- How to find the resultant force F_R from the pressure prism

$F_P = \frac{1}{2}\gamma\, h\, A = \frac{1}{2}\gamma\, h\,(bh)$ …. (3.17) (Volume of the pressure prism)

3.3.1. *Finding the point of action of F_R (the point of action) :*

From the last section , equation 3.14 :

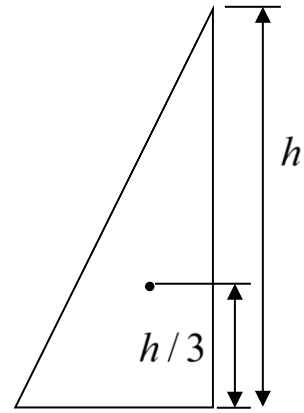
$$y_P = \frac{I_{xc}}{y_c A} + y_c = \frac{I_{xc}}{0.5\, h\,(bh)} + 0.5\, h$$

- In case of rectangular plate :

$$I_{xc} = \frac{1}{12} A h^2 = \frac{1}{12} b\, h^3 \;\text{…. (3.18)}$$

Then eq. 3.14 becomes :

$$y_P = \frac{\frac{1}{12}bh^3}{\frac{1}{2}h(bh)} + \frac{1}{2}h = \frac{1}{6}h + \frac{1}{2}h = \frac{2}{3}h \quad .... (3.19)$$ (from the top)



From the pressure prism :

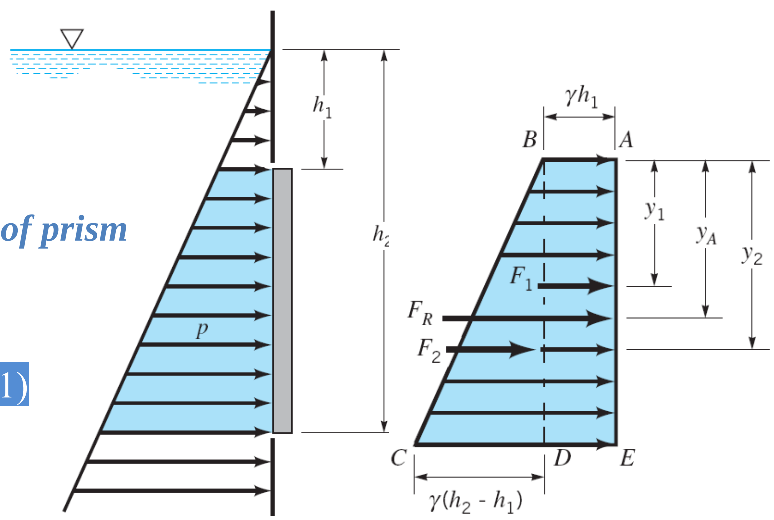
Y_P = vertical center (***Centroid of the pressure prism)***

$= \frac{2}{3}h$ (from the top) $= \frac{1}{3}h$ (above the base)

X_P = Horizontal center

Note: Special case of a plane surface not extending up to the fluid surface

- Completely submerged plane : Consider the situation shown :



1. **Resultant force F_P**

F_P = Volume of the shadow region

F_P = Volume of hexahedron + Volume of prism

$F_P = F_{1(ABDE)} + F_{2(BCD)}$ …. (3.20)

$F_P = (\gamma h_1)A + 0.5[\gamma (h_2 - h_1)] A$ …. (3.21)

2. **The location of $F_R(y_A)$: Consider the moments again**

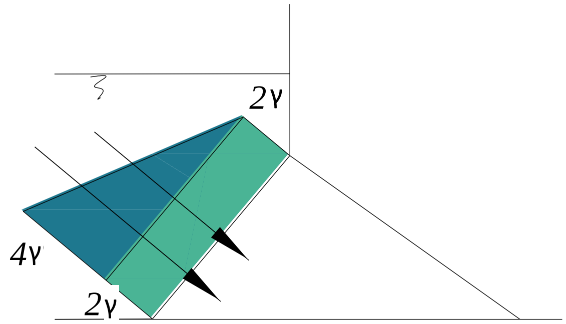
Moment by F_p acting at y_A = Moment by F_1 at y_1 + Moment by F_2 at y_2

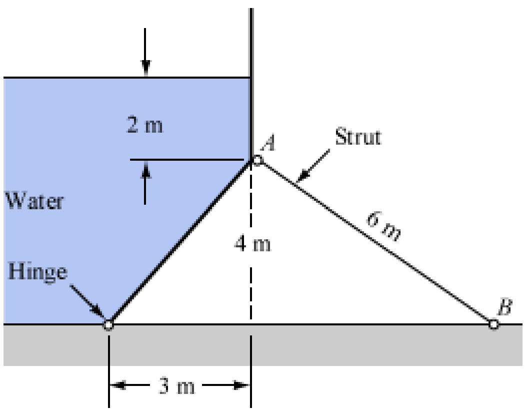
$F_P y_P = F_1 y_1 + F_2 y_2$ where $y_1 = \frac{h}{2}$ (for rectangle) $y_2 = \frac{2h}{3}$ (for triangle) (From the top)

- The effect of the atmospheric pressure p_{atm}

: Increasing Volume of hexahedron (F_1), NOT the prism (F_2)

Example 5 : ***The dam of figure has a strut AB every 6 m. Determine the compressive force in the strut, neglecting the weight of the dam.***





Solution: If the struts are doing their job, the moment about the hinge should be 0! Every six meters there is a strut, so the width each strut supports is 6 m.

Use any method to determine the forces on the dam due to the water. Using the pressure prism method:

(1) the force on the upper 2 m portion of the dam is

$$F_1 = \frac{1}{2}(2\gamma)(2\text{ m})(6\text{ m wide}) = 12\,\gamma$$

(2) the force on the lower portion of the dam is split into 2 pieces

$$F_2 = (2\gamma)(5\text{ m})(6\text{ m wide}) = 60\,\gamma$$

and

$$F_3 = \frac{1}{2}(4\gamma)(5\text{ m})(6\text{ m wide}) = 60\,\gamma$$

The compressive force on the strut AB is in a direction along the strut, and so to determine the moment about the hinge, the force is split into x- and y-components, determined by the geometry shown in Fig. 2.61,

$$F_{AB_x} = \frac{\sqrt{20}}{6} F_{AB} \qquad F_{AB_y} = \frac{4}{6} F_{AB}$$

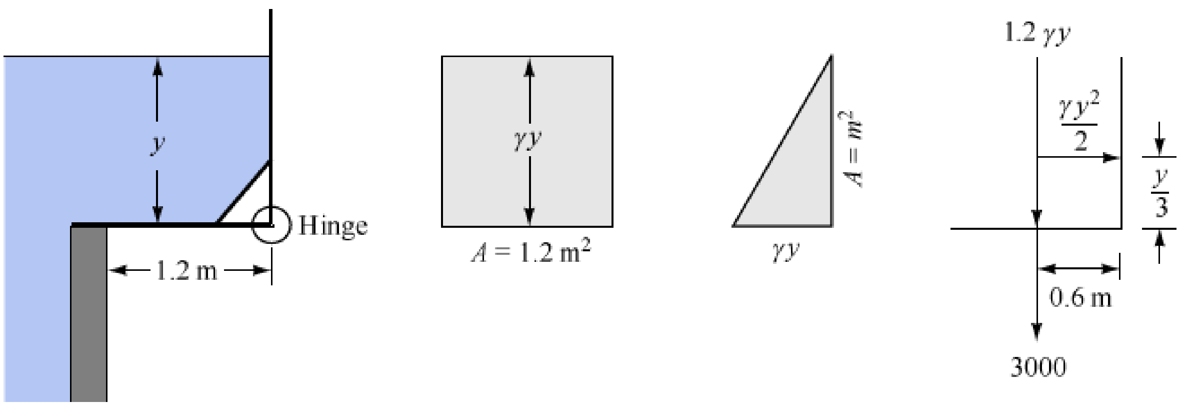
Now writing the moment equation about the hinge, (moment = force × lever arm)

$$+\circlearrowleft \Sigma M_H = 0: \quad -F_1\left[4+\frac{1}{3}(2)\right] - F_2(2.5) - F_3\left[\frac{1}{3}(5)\right] + \frac{\sqrt{20}}{6}F_{AB}(4) + \frac{4}{6}F_{AB}(3) = 0$$

or, after rearranging

$$4.9814\, F_{AB} = 549{,}136 \text{ N·m} + 1{,}470{,}900 \text{ N·m} + 980{,}600 \text{ N·m}$$
$$F_{AB} = 602{,}368 \text{ N} = 602.4 \text{ kN}$$

Example 6 : A structure is so arranged along a channel that it will spill the water out if a certain height y (Fig. 2.18a) is reached. The gate is made of steel plate weighing 2500 N/m^2. Determine the height of y.



(a) (b) (c) (d)

SOLUTION Using pressure-prism concepts, for unit width normal to the page the force on the horizontal leaf (Fig. 2.18b) is given by the volume of a pressure prism of base 1.2 m^2 and constant altitude γy N/m^2, which yields $F_y = 1.2\gamma y$ N acting through the center of the base. The pressure prism for the vertical face (Fig. 2.18c) is a wedge of base y m^2 and altitude varying from 0 to γy N/m^2. The average altitude is $\gamma y/2$, so $F_x = \gamma y^2/2$ N. The centroid of the wedge prism is $y/3$ from the hinge. The weight of the gate floor exerts a force of 3000 N at its center. Figure 2.18d shows all the forces and moment arms. For equilibrium, that is, the value of y for tipping, moments about the hinge must be zero.

$$M = (3000\ \mathrm{N})(0.6\ \mathrm{m}) + (1.2\gamma y\ \mathrm{N})(0.6\ \mathrm{m}) - \left(\frac{\gamma y^2}{2}\ \mathrm{N}\right)\left(\frac{y}{3}\ \mathrm{m}\right) = 0$$

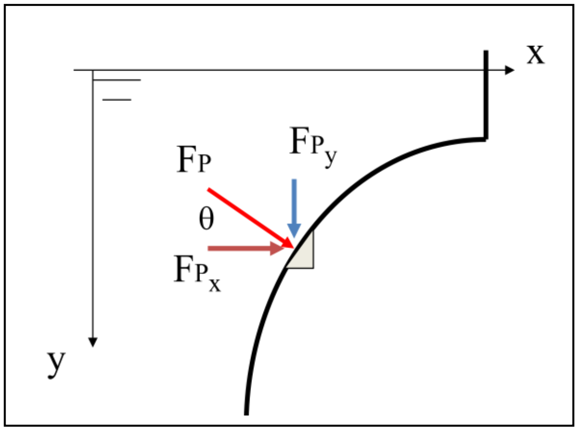
or

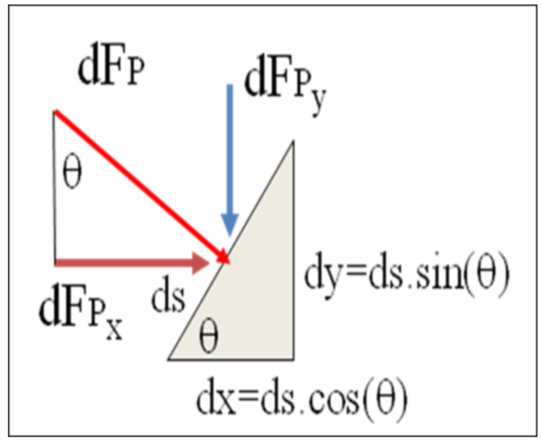
$$M = y^3 - 4.32y - 1.1014 = 0$$

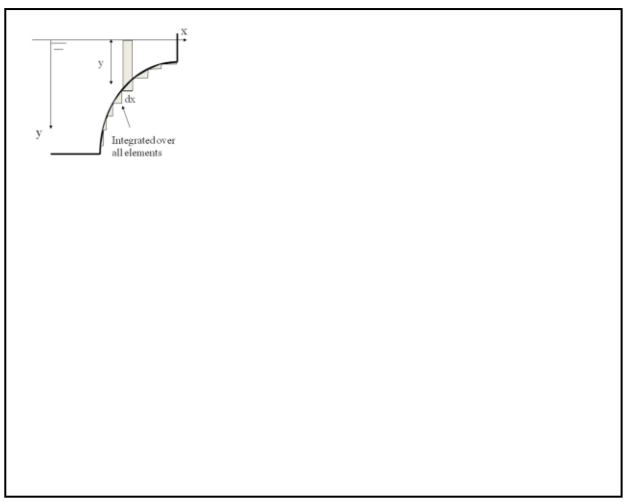
By using try and error technique (or other techniques like Newton Raphson method) we find that y=2.196m

3.4. *Hydrostatic Forces on Curved Surfaces :*

Case no.1:







$F_P = F_{py} + F_{px}$ ….(3.22)

For unit width of surface

$dF_{py} = P\ ds \cos\theta$

$P = \gamma\, y \quad$ and $\ dx = ds \cos\theta$

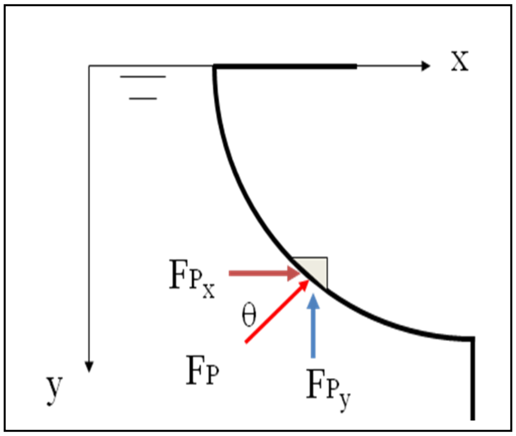
$dF_{py} = \gamma\, y\, dx$

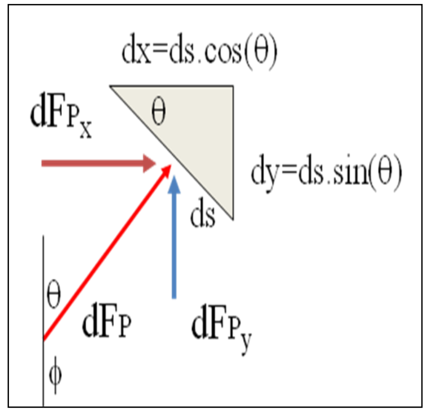
$\therefore \downarrow F_{py} = \int \gamma y\, dx = \gamma \int y\, dx = \gamma V$ …. (3.23)

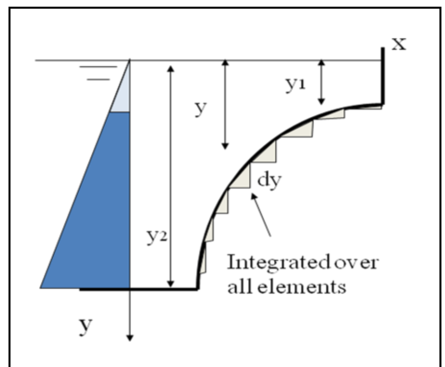
Where V the volume of liquid above the surface to the zero pressure surface

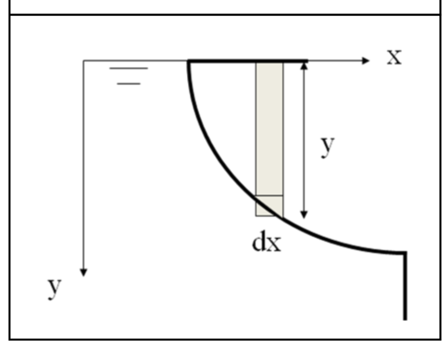
Case no.2:

By the same way we find the vertical component to pressure force if the liquid exist lower the surface by taking the sign of V as –ve to represent the upward direction of this force









$dF_{py} = P\ ds \cos \Phi = -\ P\ ds \cos \theta$

$P = \gamma\, y \qquad \& \qquad dx = ds \cos \theta$

$dF_{py} = -\ \gamma\, y\ dx$

$$\therefore \uparrow F_{py} = -\int \gamma y\, dx = -\gamma \int y\, dx = -\,\gamma\, V \ \ldots . (3.24)$$

$dF_{px} = P\ ds \sin \theta$

$P = \gamma\, y \quad \& \quad dy = ds \sin \theta$

$dF_{px} = \gamma\, y\ dy$

$$\therefore \rightarrow F_{px} = \int_{y_1}^{y_2} \gamma y\, dy = \gamma \int_{y_1}^{y_2} y\, dy = \gamma * \frac{y^2}{2}\Big]_{y_1}^{y_2}$$

$$\therefore \rightarrow F_{px} = \gamma\, \frac{y_2^2}{2} - \gamma\, \frac{y_1^2}{2} \ \ldots . (3.25)$$

Where ($\frac{\gamma y^2}{2}$) is the volume of pressure prism on the surface projection on vertical plan

The Magnitude of F_P is:

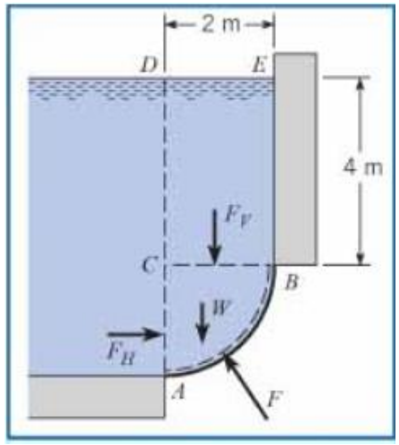
$$\therefore\ F_P = \sqrt{(F_{px})^2 + (F_{py})^2} \ \ldots . (3.26)$$ in direction of $\tan \boldsymbol{\theta} = \frac{\boldsymbol{F_{py}}}{\boldsymbol{F_{px}}}$

& the line of action can be find from the concept of:

Moment of resultant force = The summation of the moments of its components, i.e.

$$MF_p = \sum (MF_{px}, MF_{py})$$

Example 7: ***A curved surface AB is a circular arc in its section with radius of 2m and width of 1m into the paper. The distance EB is 4m. The fluid above surface AB is water, and atmospheric pressure applied on free surface of water and on the bottom side of surface AB. Find the magnitude and line of action of the hydrostatic force acting on surface AB.***



Solution:

1. Equilibrium in the horizontal direction

$F_x = F_H = P\ A = (5\ m)*(9810\ N/m^3)*(2*1\ m^2) = 98.1\ kN$

2. Equilibrium in the horizontal direction

- Vertical force on side *CB*

$$F_v = \bar{p}_0 A = 9.81\ kN/m^3 \times 4\ m \times 2\ m \times 1\ m = 78.5\ kN$$

- Weight of the water in volume *ABC*

$$W = \gamma V_{ABC} = (\gamma)(\tfrac{1}{4}\pi r^2)(w)$$
$$= (9.81\ kN/m^3) \times (0.25 \times \pi \times 4\ m^2)(1\ m) = 30.8\ kN$$

- Summing forces

$$F_y = W + F_V = 109.3\ kN$$

3. Line of action (horizontal force)

$$y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y}A} = (5\ m) + \left(\frac{1 \times 2^3/12}{5 \times 2 \times 1} m\right)$$
$$y_{cp} = 5.067\ m$$

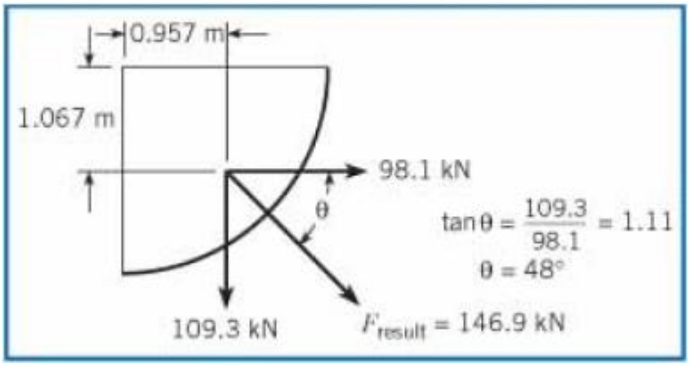
4. The line of action (x_{cp}) for the vertical force is found by summing moments about point *C*:

$$x_{cp} F_y = F_V \times 1\ m + W \times \bar{x}_W$$

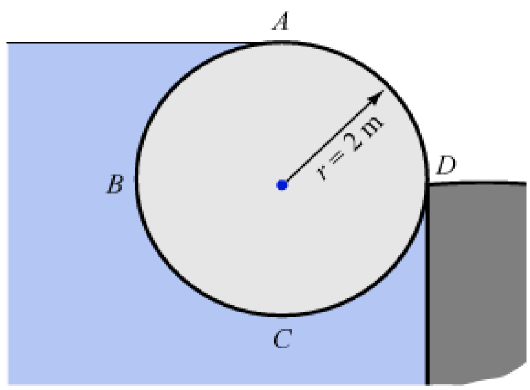
The horizontal distance from point *C* to the centroid of the area *ABC* is found using Fig. A.1: $\bar{x}_W = 4r/3\square = 0.849$ m. Thus,

$$x_{cp} = \frac{78.5\ kN \times 1\ m + 30.8\ kN \times 0.849\ m}{109.3\ kN} = 0.957\ m$$

5. The resultant force that acts on the curved surface is shown in the following figure.



Example 8: ***A cylindrical barrier in Fig. holds water as shown. The contact between the cylinder and wall is smooth. Consider a 1-m length of cylinder; determine (a) its weight, and (b) the force exerted against the wall.***



SOLUTION (a) For equilibrium the weight of the cylinder must equal the vertical component of force exerted on it by the water. (The imaginary free surface for CD is at elevation A). The vertical force on BCD is

$$F_{v_{BCD}} = \left(\frac{\pi r^2}{2} + 2r^2\right)\gamma = (2\pi + 8)\ \gamma$$

The vertical force on AB is

$$F_{v_{AB}} = -\left(r^2 - \frac{\pi r^2}{4}\right)\gamma = -(4 - \pi)\ \gamma$$

Hence, the weight per meter of length is

$$F_{v_{BCD}} + F_{v_{AB}} = (3\pi + 4)\gamma = 0.132\ \mathrm{MN}$$

(b) The force exerted against the wall is the horizontal force on ABC minus the horizontal force on CD. The horizontal components of force on BC and CD cancel; the projection of BCD on a vertical plane is zero. Hence,

$$F_H = F_{H_{AB}} = 2\gamma = 19.6\ \mathrm{kN}$$

since the projected area is 2 m^2 and the pressure at the centroid of the projected area is 9806 Pa.

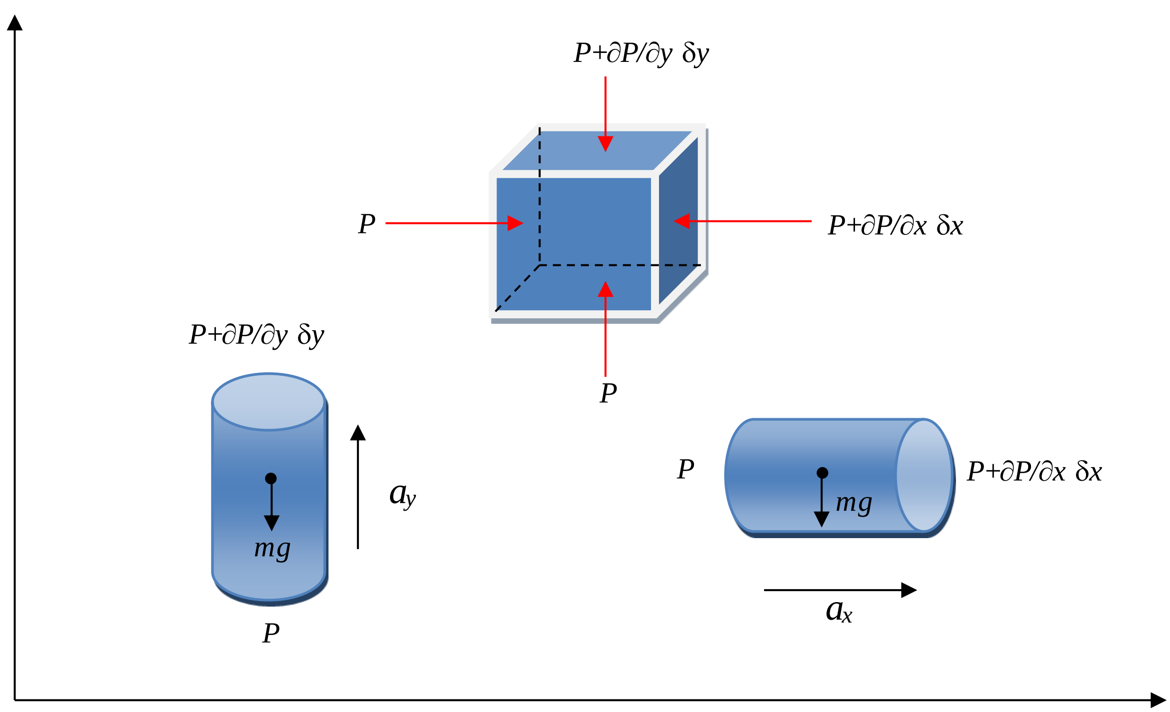
Chapter Four

ACCELERATED FLUIDS WITH

LINEAR AND ANGULAR ACCELERATION

4.1. Introduction :

If a body of fluid is moved at a constant velocity, then it obeys the equations derived earlier for static equilibrium.



If a body of fluid is accelerated such that, after some time, it has adjusted so that there are no shearing forces, there is no motion between fluid particles, and it moves as a solid block, then the pressure distribution within the fluid can be described by equations similar to those applying to static fluids.

Consider the forces acting on a small horizontal element, area δA and length δx, with a uniform

acceleration ax in the x direction:

$$F_x = \left[p\ \delta A - \left(p + \frac{\partial p}{\partial x}\ \delta x \right) \delta A \right] = -\ \frac{\partial p}{\partial x}\ \delta x\ \delta A \ldots . (4.2)$$

But Newton's 2nd law gives:

$F_x = m\ a_x$

$$\therefore\ -\ \frac{\partial p}{\partial x}\ \delta x\ \delta A = \ \rho\ \delta A\ \delta x\ a_x \ \ldots . (4.3)$$

And hence :

$$\frac{\partial p}{\partial x} = \ -\rho\ a_x \ldots . (4.4)$$

Now looking at the forces acting on a small vertical element, area δA and length δy, with a uniform acceleration az in the z direction:

$$F_y = \left[p\ \delta A - \left(p + \frac{\partial p}{\partial y}\ \delta y \right) \delta A \right] -\ \rho\ \delta y\ \delta A\ g = \ \rho\ \delta y\ \delta A\ a_y \ldots . (4.5)$$

$$\therefore\ -\ \frac{\partial p}{\partial y}\ \delta y\ \delta A -\ \rho\ \delta y\ \delta A\ g = \ \rho\ \delta A\ \delta y\ a_y \ \ldots . (4.6)$$

And hence :

$$\frac{\partial p}{\partial y} = \ -\rho\ (g + a_y) \ldots . (4.7)$$

Here, pressure p, is a function of x and y, so, by definition:

$$dp = \ \frac{\partial p}{\partial x}\ \delta x\ +\ \frac{\partial p}{\partial y}\ \delta y \ldots . (4.8)$$

and identifying isobars as lines of constant pressure with dp = 0, we have an equation for the slope of an isobar as:

$$\frac{\partial p}{\partial x} = \ \frac{\partial p}{\partial y} . \frac{dy}{dx} \ldots . (4.9)$$

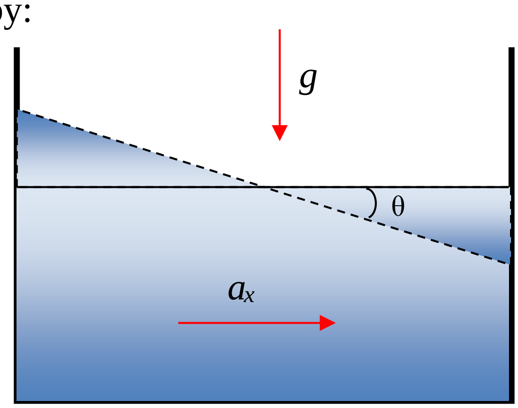
Then :

$$\frac{dy}{dx} = \frac{\frac{\partial p}{\partial x}}{\frac{\partial p}{\partial y}} = \frac{\rho\, a_x}{\rho\,(g+a_y)} \quad \ldots .(4.10)$$

It is clear that if $a_x \neq 0$, then the isobars will not be horizontal in this case.

4.2. Uniform Linear Acceleration

If a container of fluid is accelerated uniformly horizontally with $a_y = 0$, then the slope of the isobars within the fluid is given by:



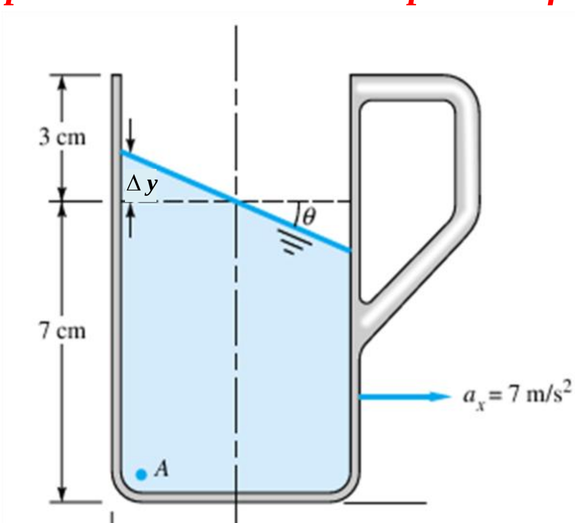
$$\theta = \tan^{-1}\frac{a_x}{(g+a_y)} \quad \ldots .(4.11)$$

Notes :

- hydrostatic pressure variation in a gas is negligible in comparison to that in a liquid,
- the free surface of a liquid is normally taken as a line of constant pressure - or isobar - and the equation above gives the surface slope of an accelerated fluid.
- The resultant acceleration perpendicular on the isobar surfaces is:

$$G = \sqrt{{a_x}^2 + (g + a_y)^2} \quad \ldots .(4.12)$$

Example (1) : ***A drag racer rests her coffee mug on a horizontal tray while she accelerate at 7 m/s2. The mug is 10 cm deep and 6 cm in diameter and contains coffee 7 cm deep at rest. (a) Assuming rigid body acceleration of the coffee, determine whether it will spill out of the mug.(b) calculate the gage pressure in the corner at point A if the density of coffee is 1010 kg/m3.***



Solution :

Part a : The free surface tilts at the angle θ gi 54 eq. 4.11 regardless of the shape of the mug. With $a_y = 0$ and standard gravity :

$$\theta = tan^{-1}\frac{a_x}{g} = tan^{-1}\frac{7}{9.81} = 35.5^o$$

If the mug is symmetric about its central axis, the volume of coffee is conserved if the tilted surface exactly at the centerline, as shown in figure above :

Thus the deflection at the left side of the mug is :

$y = 3 * \tan\theta = 2.14$ cm *Ans. (a)*

this is less than the 3 cm clearance available, so the coffee will not spill unless it was sloshed during the start- up of acceleration.

Part a : When at the rest, the gage pressure at point A is given by :

$P_A = \rho\, g\, (y_{surface} - y_A) = 1010 * 9.81 * 0.07 = 694\ N/m^2 = 694$ Pa.

During acceleration, eq. 4.12 applies, with :

$G = [(7)^2 + (9.81)^2\ ]^{0.5} = 12.05\ m/s^2$

The distance Δs down the normal from the tilted surface to point A is :

$\Delta s = (\ 7 + 2.14\ ) \cos\theta = 7.44$ cm.

Thus the pressure at point A becomes:

$P_A = \rho G\, \Delta s = 1010 * 12.05 * 0.0744 = 906$ Pa *Ans. (b)*

Which is an increase of 31% over the pressure when at rest.

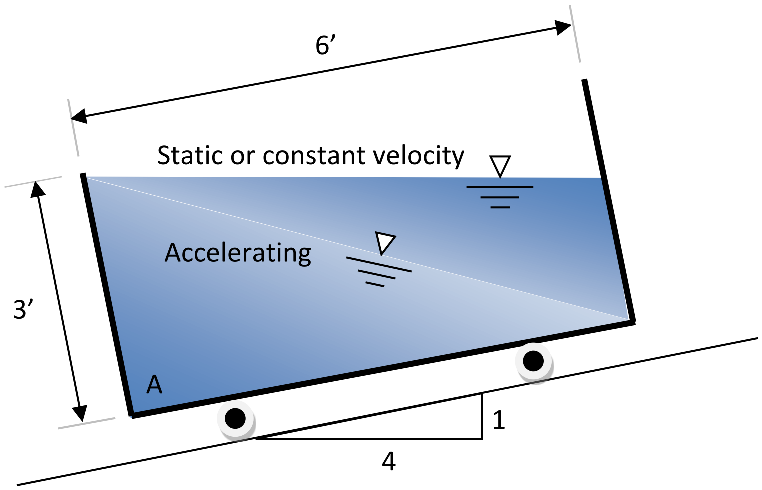
***Example (2) :* An open tank of water is accelerated vertically upward at 15 ft/sec^2. Calculate the pressure at a depth of 5 ft.**

Solution :

$$\frac{dp}{dz} = -62.4\left(\frac{15+32.2}{32.2}\right) = -91.5\ \text{lb/ft}^3$$

$$p = \int_0^p dp = -\int_0^{-5} 91.5\ dz = 457\ \text{psf}$$

Example (3) : ***This open tank moves up the 55 with constant acceleration .Calculate the acceleration required for the water surface to move to the position indicated. Calculate the pressure in the corner of the tank at A before and after acceleration.***



Solution :

From geometry, the slope of the water surface during acceleration is -0.229. from the slope of the plane, $a_x = 4\ a_z$:

$$-0.229 = \frac{-4\ a_z}{(a_z + 32.2)}$$

And from the foregoing :

$a_z = 1.96\ \text{ft/sec}^2$, $a_x = 7.84\ \text{ft/sec}^2$, $a = 8.08\ \text{ft/sec}^2$

from geometry, the depth of water vertically above corner A before acceleration is 2.91 ft ; hence the pressure there is 2.91 * 62.4 = 181.5 psf. After acceleration this depth is 2.75 ft :

$$\frac{\partial p}{\partial z} = -(32.2 + 1.96)\left(\frac{62.4}{32.2}\right) = -66.0\ lb/ft^3$$

Therefore :

P = 2.75 * 66 = 181.5 psf.

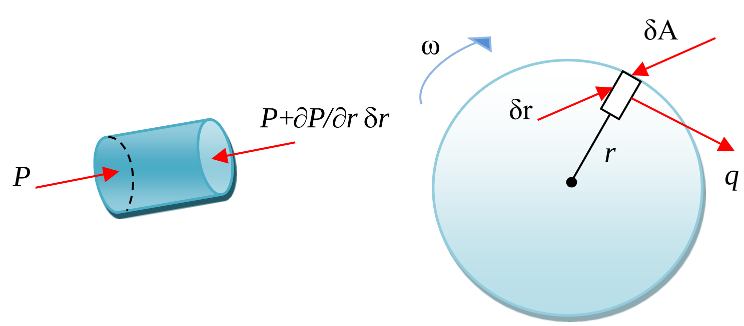
Note:

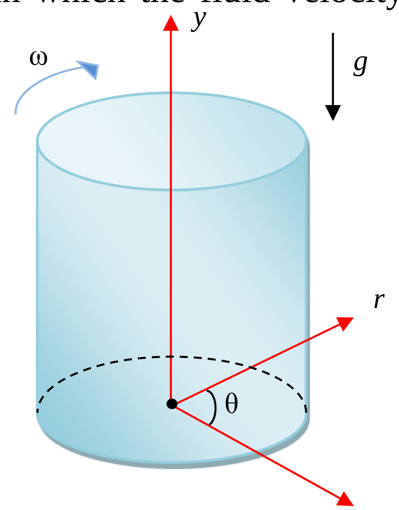
The fact that the pressure at point A are the same before and after acceleration is no coincidence; general proof may be offered that p_A does not change whatever the acceleration. This means that the force exerted by the end of the tank on the water is constant for all accelerations. However,

this is no violation of Newton’s second law , since the mass of liquid diminishes with increased acceleration so that the product of mass and acceleration remains constant and equal to the applied force.

4.3. Uniform Angular Acceleration(Rotation about Vertical Axis):

If liquid is placed in a container and rotated about a vertical axis at a constant angular velocity, then after some time it will move as a solid body with no shearing of the fluid. Under such conditions the liquid is said to be moving as a "***forced vortex***" with velocity $q = \omega r$ at any radius r from the axis. [This contrasts with "***free-vortex***" motion in which the fluid velocity varies inversely with distance from the axis of rotation].





With ω constant, all fluid experiences an acceleration $\omega^2 r$ (centrifugal) directed towards the axis of rotation, and for equilibrium of a typical small horizontal element:

$$\frac{\partial p}{\partial r}\delta r\ \delta A = \rho\ \delta A\ \delta r\ \omega^2 r \ \ldots.(4.13)$$

$$\therefore \frac{\partial p}{\partial r} = \rho\ \omega^2 r \equiv \rho\ \frac{q^2}{r} \ldots.(4.14)$$

In the vertical direction, the usual expression for pressure distribution in a static fluid holds:

$$\frac{\partial p}{\partial y} = -\rho\ g \ \ldots.(4.15)$$

Clearly, in such circumstances, the pressure is varying with both r and y: p = f(r, y),and

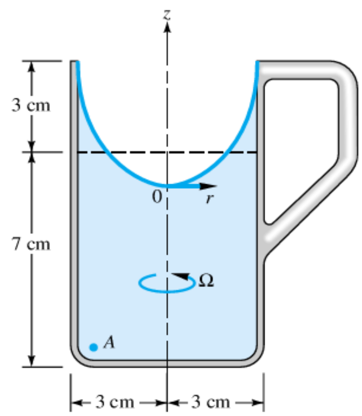
$$dp = \frac{\partial p}{\partial r}dr + \frac{\partial p}{\partial y}dy \ \ldots.(4.16)$$

Hence: $dp = \rho\ \omega^2 r\ dr - \rho g\ dy$, and if using a liquid with constant □, this can be integrated:

$$p = \frac{1}{2}\ \rho\ \omega^2 r^2 - \rho g y + constant \ \ldots.(4.17)$$

- This reveals that all isobars in such a rotating liquid are paraboloids with the form: $r^2 = k\,y + constant$
  - and the free surface of the liquid (being an isobar) will also take this form.
  - We can find the equation of each isobar lines by applying the boundary condition of the center of rotation which have the coordinate of $(0, y_o)$.

Example (4) : ***The coffee up in example 1 is removed from the drag racer, placed on a turntable, and rotated about its central axis until a rigi y mode occurs. Find (a) the angular velocity which will cause the coffee to just reach the the cup and (b) the gage pressure at point A for this condition.***



Solution :

The cup contains 7 cm of coffee. The remaining distance of 3 cm up to the lip must equal the distance $h/2$ in Fig. 2.23. Thus

$$\frac{h}{2} = 0.03\text{ m} = \frac{\Omega^2 R^2}{4g} = \frac{\Omega^2 (0.03\text{ m})^2}{4(9.81\text{ m/s}^2)}$$

Solving, we obtain

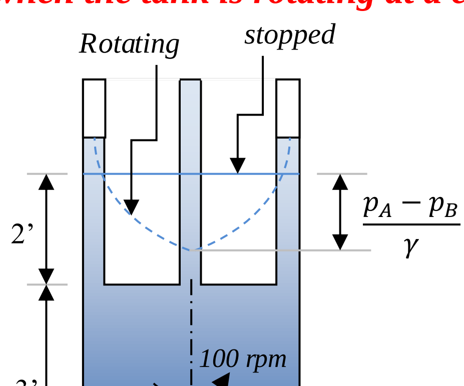
$$\Omega^2 = 1308 \qquad \text{or} \qquad \Omega = 36.2\text{ rad/s} = 345\text{ r/min} \qquad \textit{Ans. (a)}$$

To compute the pressure, it is convenient to put the origin of coordinates r and z at the bottom of the free-surface depression, as shown in Fig. E2.13. The gage pressure here is $p_0 = 0$, and point A is at $(r, z) = (3\text{ cm}, -4\text{ cm})$. Equation (2.63) can then be evaluated

$$\begin{aligned} p_A &= 0 - (1010\text{ kg/m}^3)(9.81\text{ m/s}^2)(-0.04\text{ m}) \\ &\quad + \tfrac{1}{2}(1010\text{ kg/m}^3)(0.03\text{ m})^2(1308\text{ rad}^2/\text{s}^2) \\ &= 396\text{ N/m}^2 + 594\text{ N/m}^2 = 990\text{ Pa} \qquad \textit{Ans. (b)} \end{aligned}$$

This is about 43 percent greater than the still-water pressure $p_A = 694$ Pa.

Example (5) : ***This tank is fitted with three piezometer columns of the same diameter. Calculate the pressure heads at points A & B when the tank is rotating at a constant speed of 100 rpm.***



Solution :

During rotation the surface in the piezometer column must be on a parabola of constant pressure, and liquid lost from the central column must appear in the other two. Accordingly :

$$\frac{p_B - p_A}{\gamma} = \frac{(2\pi * \frac{100}{60})^2 \, (1)^2}{2g} = 1.71 \text{ ft}$$

$$\frac{p_A}{\gamma} = 5 - \frac{2}{3}\,(1.71) = 3.86 \text{ ft}$$

$$\frac{p_B}{\gamma} = 5 + \frac{1}{3}\,(1.71) = 5.57 \text{ ft}$$

Example (6) : ***In a laboratory experiment, a circular tank of radius r_o = 2 m is filled with water to an average depth d = 75 mm and rotated with an angular speed ω = 5 rpm about its center. Find the shape of the free surface of the water when it is in rigid body rotation.***

Solution :

The free surface is a parabola

$z - z_o = \omega^2 r^2 / 2g$

with z_o = the presently unknown depth at r = 0. The mass of fluid in the tank must be conserved so :

$$\rho \pi r_o^{\,2} d = \rho \int_0^{t_o} 2\pi \, r \, z \, dr = \frac{2\pi\rho\omega^2}{2g} \int_0^{t_o} r^3 dr + 2\pi\rho z_o \int_0^{t_o} r \, dr$$

$$r_o^{\,2} \, d = \frac{\omega^2 r_o^{\,4}}{4g} + r_o^{\,2} z_o$$

And :

$$z_o = d - \frac{\omega^2 r_o^{\,2}}{4g} = 0.075 - (\frac{2\pi * 5}{60})^2 \frac{4}{4 * 9.81} = 0.047 \, m = 47 \, mm$$

Thus :

$z = 0.047 + 0.014\, r^2 \leq 0.103 = 103$ mm

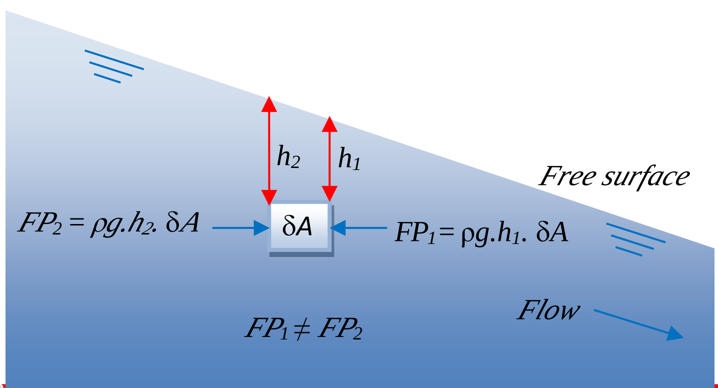
that is, the water depth z ranges from 47 mm at the tank center to 103 mm at the tank wall. Compare these differences to those in the confined tank results above. Are the same principles operative?

Chapter Five

TYPES OF FLOW (FLUID FLOW)

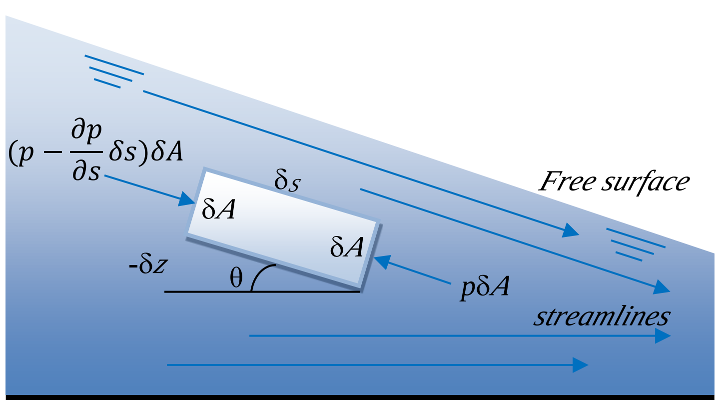
5.1. Introduction :

Motion (flowing) of a fluid subjected to unbalanced forces or stresses that caused if the fluid mass was subjected to hydraulic gradient (e.g. tilting of free surface by certain angle or connect two containers have different levels). The motion continues as long as unbalanced forces are applied.



5.2. Ideal Fluid Flow (Derivation of Euler's Equation & Bernoulli's Equation):

Consider a small element of fluid aligned along a streamline. It has a cross sectional area δA, pressure is assumed uniform across its ends δA, and the local velocity is defined v.



Applying Newton's laws of motion to the flow through the element along the streamline:

The force (in the direction of motion along the streamline) = mass x acceleration.

1. First we look at the acceleration of the fluid element.
   - Ignoring the possibility that the flow might be steady, $\frac{\partial v}{\partial t} \neq 0$
   - ***v*** can change with time ***t***, and also with position ***s*** along the streamline.
     i.e. $\boldsymbol{v = f\,(t, s)}$.
   - Hence, if the element moves a distance δ***s*** in time δ***t***, then the total change in velocity δ***v*** is given by:

$$\delta v = \frac{\partial v}{\partial s}\delta s + \frac{\partial v}{\partial t}\delta t \;....(5.1)$$

and in the limit as δ***t*** tends to zero, the "substantive" derivative is given as:

$$\frac{dv}{dt} = \lim_{\delta t \to 0}\frac{\delta v}{\delta t} = \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = v\underbrace{\frac{\partial v}{\partial s}}_{\text{spatially}} + \underbrace{\frac{\partial v}{\partial t}}_{\text{temporarily}} \;....(5.2)$$

   - for a steady flow the local velocity at a point ($\frac{\partial v}{\partial t}$) does not vary with time, so the last term under such circumstances will be zero.

2. Looking now at the forces acting on the element and applying Newton's 2nd law:

$$\left(p - \frac{\partial p}{\partial s}\delta s\right)\delta A - p\delta A + \rho\delta A\,\delta s\,g\sin\theta = \rho\,\delta s\,\delta A\,v\,\frac{dv}{ds} \;....(5.3)$$

dividing through by ***δA. δs*** and defining ***-δz = δs sin θ*** , we have that:

$$\frac{\partial p}{\partial s} + \rho v\frac{\partial v}{\partial s} + \rho g\,\frac{\delta z}{\delta s} = 0 \;....(5.4)$$

and in the limit as ***δs* tends to zero,**

$$\frac{dp}{ds} + \rho v\,\frac{dv}{ds} + \rho g\,\frac{dz}{ds} = 0 \;....(5.5)$$

$$\frac{dp}{\rho} + v\,dv + g\,dz = 0 \;....(5.6)$$

Euler's equation (for ideal, steady flow)

Or

$$\frac{dp}{\rho} + v\,dv + \;g\,dz = 0 \;....(5.7)$$

This is a form of ***Euler's equation***, and *p*, *v*, and *z* in flow field.

- it then becomes possible to integrate it - giving:

$$\frac{p}{\rho} + \frac{1}{2}v^2 + gz = C \;....(5.8)$$

$$p + \frac{1}{2}\,\rho v^2 + \rho g z = C \;....(5.9)$$

Bernoulli's equation (for ideal, steady flow)

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = C \;....(5.10)$$

The three equations above are valid for incompressible, frictionless steady flow, and what they state is that total energy is conserved along a streamline.

The first of these forms of the ***Bernoulli equation*** is a measure of energy per unit mass, the second of energy per unit volume, and the third of "head", equivalent to energy per unit weight.

In the second equation, the term ***p*** is the static pressure, **{½ρv^2}** is the dynamic pressure, ***ρgz*** is the elevational term, and the **SUM** of all three is known as the **stagnation (or total) pressure, p_0**

In the third equation, ***p/ρg*** is known as the pressure head (or flow work head or flow energy head), which is the work done to move fluid against pressure, ***v^2/2g*** as the kinetic head (dynamic energy head or velocity head), ***z*** is the potential head (elevation head) and the sum of the three terms as the **Total Head *H***. The sum of first and third tem of 3rd equation is called the piezometric head respect to piezometer's tube.

where **C** is a constant along a streamline.

- For the special case of irrotational flow, the constant C is the same everywhere in the flow field.
- Therefore, the Bernoulli equation can be applied between any two points in the flow field if the flow is **ideal, steady, incompressible**, and **irrotational**.

- i.e. for two points 1 and 2 in the flow field:

$$\frac{p_1}{\rho g}+\frac{{v_1}^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{{v_2}^2}{2g}+z_2 \;....(5.11)$$

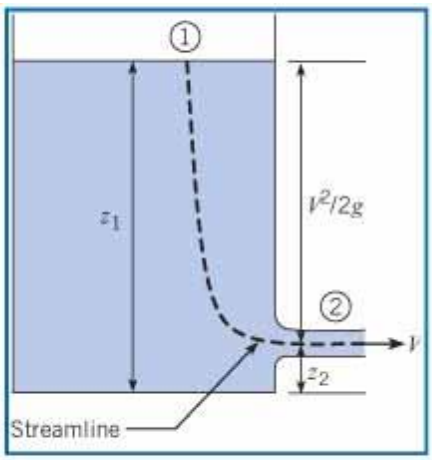
Example (1): ***An open tank filled with water ar 62 ins through a port at the bottom of the tank. The elevation of the water in the tank is 10 m above the drain. The drain port is at atmospheric pressure. Find the velocity of the liquid in the drain port.***

Assumptions:

1. Flow is steady.

2. Viscous effects are unimportant.

3. Velocity at liquid surface is much less than velocity in drain port.



Solution:

1. The Bernoulli equation between points 1 and 2 on streamline:

$$\frac{p_1}{\rho g}+\frac{{v_1}^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{{v_2}^2}{2g}+z_2$$

2. The pressure at the outlet and the tank surface are the same (atmospheric), so $p_1 = p_2$. The velocity at the tank surface is much less than in the drain port so $V_1 >> V_2$. Solution for V_2:

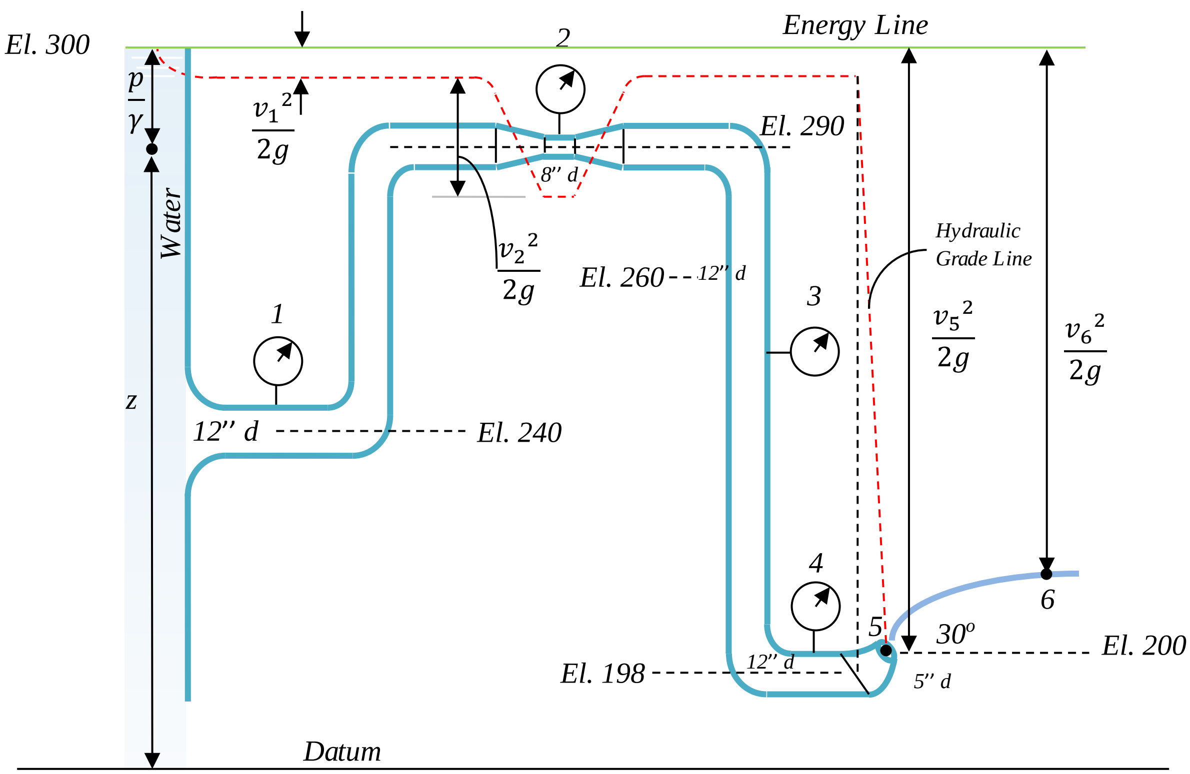
$$0+0+(z_1-z_2)=0+\frac{{v_2}^2}{2g}$$

3. Velocity calculation

$$V_2=\sqrt{2g(z_1-z_2)}=\sqrt{2gH}=\sqrt{2\times 9.81\times 10}=14m/s$$

Which states that the speed of efflux is equal to the speed of free fall from the surface of the reservoir. This is known as ***Torricelli's theorem***.

Example (2): ***Calculate the flow rate through this pipeline and nozzle. Calculate the pressure at points 1, 2, 3 and 4 in the pipe and the elevation of the top of the jet's trajectory.***



Solution:

First sketch the energy line; it is evident that, at all points in the reservoir where the velocity is negligible, (z + p/γ) will be the same. Thus the energy line has the same elevation as the water surface. Next sketch the hydraulic grade line; this is coincident with the energy line in the reservoir where the velocity is negligible but drops below the energy line over the pipe entrance where the velocity is gained. The velocity in the 12- in. Pipe is everywhere the same, so the hydraulic grade line must be horizontal until the flow encounters the constriction upstream from section 2. Here, as velocity increases, the hydraulic grade line must fall (possibly to a level below the constriction). Downstream from the constriction the hydraulic grade line must rise to the original level over the 12- in. Pipe and continue at this level to a point over the base of the nozzle

at section 4. Over the nozzle the hydraulic grade line must fall to the nozzle tip and after that follow the jet, because the pressure in the jet is everywhere zero.

Since the vertical distance between the energy line and the hydraulic grade line at any point is the velocity head at that point, it is evident that:

$$\frac{{v_5}^2}{2g} = 100 \qquad and\ therefore \quad v_5 = 80.2\ fps$$

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Thus :

$$Q = 80.2 * \frac{\pi}{4}\left(\frac{5}{12}\right)^2 = 10.95\ cfs$$

From continuity considerations,

$$v_1 = \left(\frac{5}{12}\right)^2 v_5$$

And thus :

$$\frac{{v_1}^2}{2g} = \left(\frac{5}{12}\right)^4 \frac{{v_5}^2}{2g} = \left(\frac{5}{12}\right)^4 100 = 3\ ft$$

Similarly :

$$\frac{{v_2}^2}{2g} = \left(\frac{5}{8}\right)^4 100 = 15.3\ ft$$

Since the pressure heads are conventionally taken as the vertical distances between the pipe center line and hydraulic grade line, the pressure in pipe and constriction may be computed as follows:

$p_1/\gamma = 60 - 3 = 57$ ft ; $p_1 = 24.7$ psi

$p_2/\gamma = 10 - 15.3 = -5.3$ ft ; $p_2 = 4.7$ in. Hg vacuum

$p_3/\gamma = 40 - 3 = 37$ ft ; $p_3 = 16$ psi

$p_4/\gamma = 102 - 3 = 99$ ft ; $p_4 = 42.8$ psi

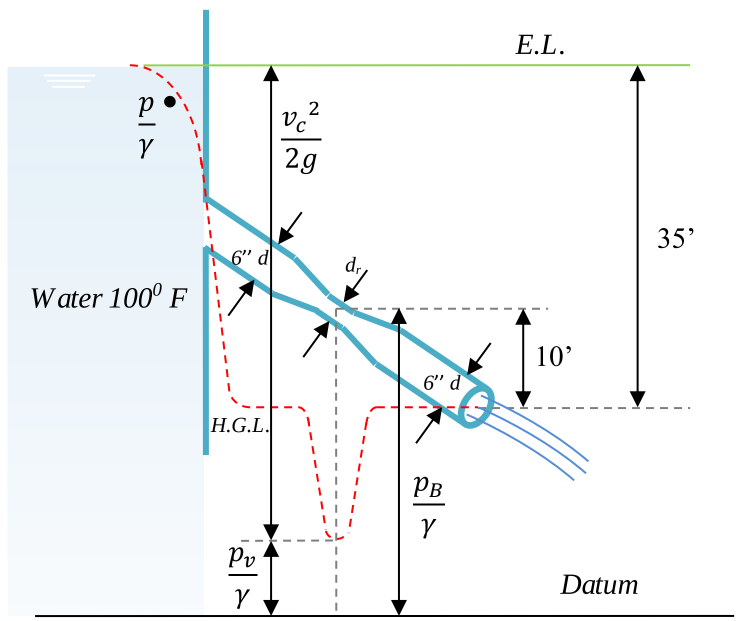
The velocity of the jet at the top of its trajectory (where there is no vertical component of velocity) is given by :

V_6 = 80.2 cos 30° = 69.5 fps

And the elevation here is :

300 – (69.5)^2 / 2g = 225 ft.

Example (3): ***The barometric pressure is 14 psia. What diameter of constriction can be expected to produce incipient cavitation at the throat of the constriction ?***



Solution :

From tables (appendix 2 : physical properties of water (Elementary fluid mechanics by Vennard and Streeter)

γ = 62 lb/ft^3 and p_v / γ = 2.2 ft (at temperature of 100° F)

p_B / γ = (14*144)/62 = 32.5 ft.

Construct energy line and hydraulic grade line as indicated on the sketch. Evidently the velocity head at the constriction is given by :

$$\frac{v_c{}^2}{2g} = 35 - 10 + 32.5 - 2.2 = 55.3\ ft.$$

$$\frac{v_6{}^2}{2g} = 35\ ft$$

Since, from continuity considerations :

$(\frac{d_c}{6})^2 = \frac{v_6}{v_c}$, $(\frac{d_c}{6})^4 = (\frac{v_6}{v_c})^2 = \frac{35}{55.3}$

$d_c = 5.35$ inch

5.3. Flow Classification (Flow pattern)

5.3.1.Uniform or Non Uniform Flow:

Uniform flow : is a flow in which the velocity does not change along a streamline, i.e.

$$\frac{\partial v}{\partial s} = 0$$

In uniform flows the streamlines are straight and parallel .

Nonuniform flow: is a flow in which the velocity changes along a streamline, i.e.

$$\frac{\partial v}{\partial s} \neq 0$$

5.3.2.Steady or Unsteady Flow:

Steady flow: is a flow in which the velocity at a given point on a streamline does not change with time: $\frac{\partial v}{\partial t} = 0$

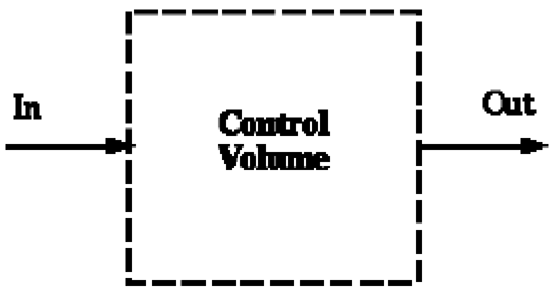
Unsteady flow: exists if: $\frac{\partial v}{\partial t} \neq 0$

Combining the above we can classify any flow in to one of four types:

- ***Steady uniform flow.*** Conditions do not change with position in the stream or with time. **An example** is the flow of water in a pipe of constant diameter at constant velocity.
- ***Steady non-uniform flow.*** Conditions change from point to point in the stream but do not change with time. **An example** is flow in a tapering pipe with constant velocity at the inlet - velocity will change as you move along the length of the pipe toward the exit.
- ***Unsteady uniform flow.*** At a given instant in time the conditions at every point are the same, but will change with time. **An example** is a pipe of constant diameter connected to a pump pumping at a constant rate which is then switched off.
- ***Unsteady non-uniform flow.*** Every condition of the flow may change from point to point and with time at every point. **An example** is surface waves in an open channel.

5.4. Control Volume

A control volume is a finite region, chosen by the analyst for a particular problem, with open boundaries through which mass, momentum, and energy are allowed to cross. The analyst makes a budget, or balance, between the incoming and



outgoing fluid and the resultant changes within the control volume. Therefore one can calculate the gross properties (net force, total power output, total heat transfer, etc.) with this method.

With this method, however, we do not care about the details inside the control volume (In other words we can treat the control volume as a "black box.")

5.5. Flow Rate (Discharge)

$$Mass\ Flow\ Rate = \frac{Mass}{Time\ taken\ to\ accumulative\ this\ mass} = \rho\ .volume\ of\ flow\ rate = Discharge\ (Q) \ldots.(5.12)$$

Volume flow rate (Discharge) :

$$Volume\ Flow\ Rate = \frac{Volume}{Time\ taken\ to\ accumulative\ this\ volume} = \ \ldots.(5.13)$$

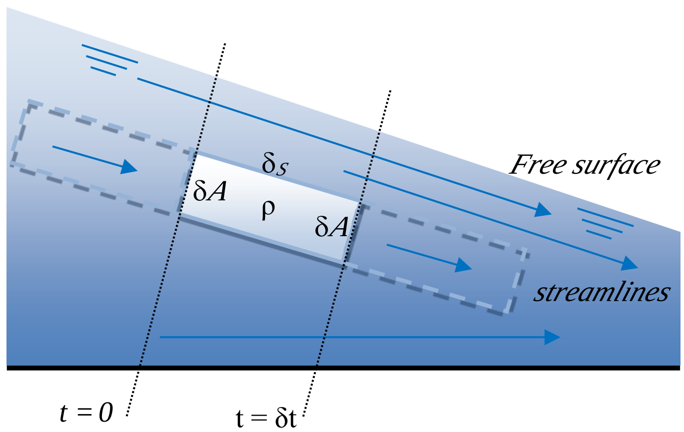
If we need to calculate the discharge by knowing the ***cross sectional area of a pipe is A*** and

Also we must know the ***Mean velocity*** is V_m.

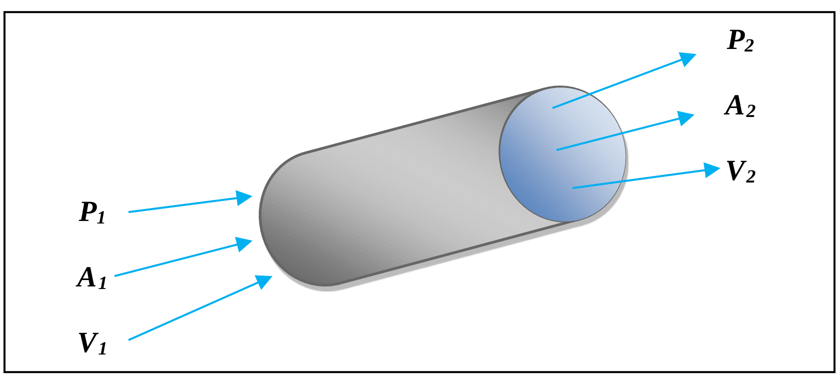
$Q = A\ .\ V_m \ldots.(5.14)$

5.6. Continuity Equation (Conservation of Mass)

Mass entering per unit time = Mass leaving per unit time + Change of mass in control volume per unit time



Applying to a stream tube as shown in figure below :



Mass entering per unit time = Mass leaving per unit time

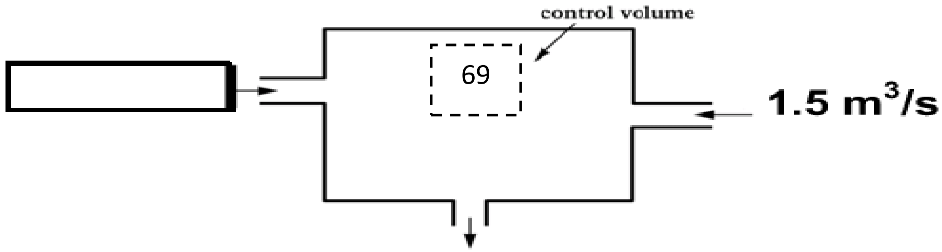
$\rho_1\ \delta A_1\ V_1 = \rho_2\ \delta A_2\ V_2$ …. (5.15)

or for steady flow :

$\rho_1\ \delta A_1\ V_1 = \rho_2\ \delta A_2\ V_2\ = \text{constant} = m = \frac{dm}{dt}$ ….(5.16) **Continuity equation**

to explain all above principles , the following example will show how to compute the inflow and outflow

1. What is the outflow?

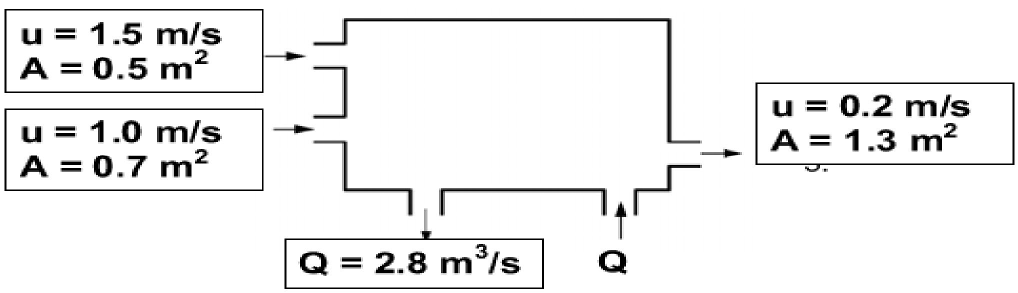


$Q_{in} = Q_{out}$

$1.5 + 1.5 = 3$

$Q_{out} = 3.0\ m^3/s$

2. What is the inflow?



Q = Area × Mean Velocity = Au

$Q + 1.5\times0.5 + 1\times0.7 = 0.2\times1.3 + 2.8$

Q = 3.72 m3/s

Example (4): ***Calculate the flow rate of gasoline (s.g. = 0.82) through this pipe line, using first the gage readings and then the manometer reading ?***

Solution :

With sizes of pipe and constriction given, the problem centers around the computation of the value of the quantity ($p_1/\gamma + z_1 - p_2/\gamma - z_2$) of the Bernoulli equation.

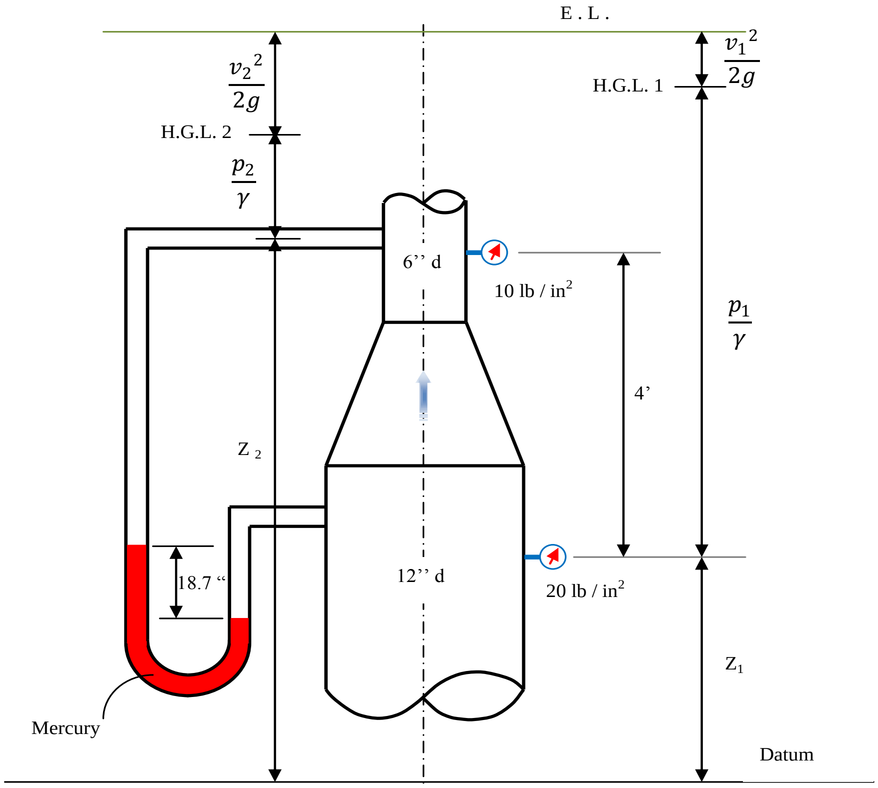
Taking datum at the lower gage and using the gage reading,

$$\left(\frac{p_1}{\gamma} - z_1 - \frac{p_2}{\gamma} - z_2\right) = \frac{20 * 144}{0.82 * 62.4} - \frac{10 * 144}{0.82 * 62.4} - 4 = 24.2 \quad \text{ft of gasoline}$$

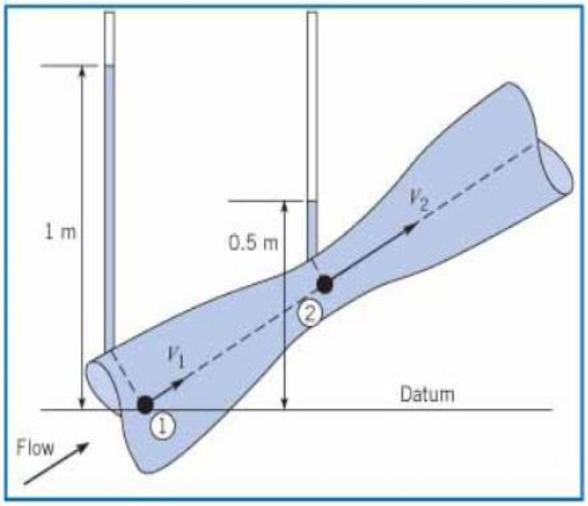
To use the manometer reading, construct the hydraulic grade line levels at 1 and 2. It is evident at once that the difference in these levels is the quantity ($p_1/\gamma + z_1 - p_2/\gamma - z_2$) and, visualizing ($p_1/\gamma$) and ($p_2/\gamma$) as liquid columns, it is also apparent that the difference in the hydraulic grade line levels is equivalent to the manometer reading. Therefore :

$$\left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2\right) = \frac{18.7}{12}\left(\frac{13.57-0.82}{0.82}\right) = 24.2 \text{ ft of gasoline}$$

Insertion of 24.2 ft into the Bernoulli equation followed by its simultaneous solution with the continuity equation yields a flowrate Q of 8.0



Example (5) : (Venturi Section) ***Piezometric tubes are tapped into a Venturi section as shown in the figure. The liquid is incompressible. The upstream piezometric head is 1 m, and the piezometric head at the throat is 0.5 m. The throat section is twice large as in the approach section. Find the velocity in the throat section.***



Problem Definition

Situation: Incompressible flow in venturi section ometric heads and sections ratio given.

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Find: Velocity (in m/s) in venturi section.

Assumptions: Viscosity effects are negligible, and the Bernoulli equation is applicable.

Plan:

1. Write out the continuity equation, incorporating sections ratio:
2. Write out the Bernoulli equation, incorporating continuity eq. and solve for throat velocity.
3. Substitute in piezometric heads to calculate throat velocity.

Solution :

The continuity equation Bernoulli gives:

$$v_1 \cdot A_1 = v_2 \cdot A_2$$
$$A_1 = 2A_2$$
$$v_2 = 2v_1$$

The Bernoulli equation with $v_2 = 2v_1$ gives

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\frac{p_1}{\rho g}+z_1-\left(\frac{p_2}{\rho g}+z_2\right)=\frac{v_2^2}{2g}-\frac{v_1^2}{2g}=\frac{3\cdot v_1^2}{2g}$$

$$1.0-0.5=\frac{3}{2g}\times v_1^2$$

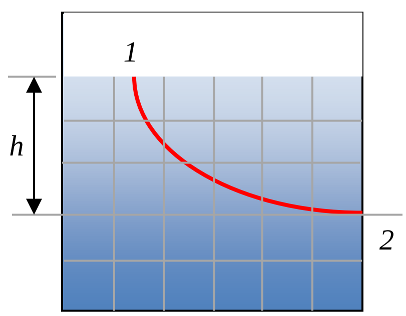
$$v_1=\sqrt{\frac{2g}{3}\times 0.5}=1.81\,m/s$$

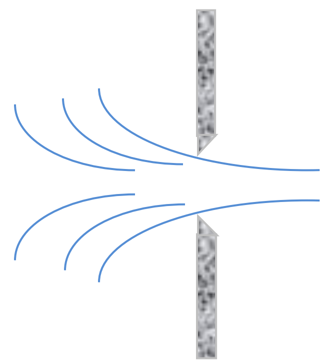
$$v_2=2\times 1.81=3.62\,m/s$$

5.7. Applications of Bernoulli's and Continuity Equations :

5.7.1. Flow from Tanks (Flow Through A Small Orifice)

Flow from a tank through a hole in the side :





The edges of the hole are sharp to minimize frictional losses by minimizing the contact between the hole and the liquid.

The streamlines at the orifice contract reducing the area of flow.

This contraction is called the vena contracta

The amount of contraction must be known to calculate the flow

Apply Bernoulli's eq. along the streamline joining point 1 on the surface to point 2 at the centre of the orifice.

At the surface velocity is negligible ($u_1 = 0$) and the pressure atmospheric ($p_1 = 0$).

At the orifice the jet is open to the air so again the pressure is atmospheric ($p_2 = 0$).

If we take the datum line through the orifice, then $z_1 = h$ and $z_2 = 0$, leaving

$$\boldsymbol{h = \frac{v_2{}^2}{2g}} \;, \qquad \textbf{and} \qquad \boldsymbol{v_2 = \sqrt{2gh}}$$

This theoretical value of velocity is an overestimate as friction losses have not been taken into account.

A coefficient of velocity is used to correct the theoretical velocity,

$$\boldsymbol{V_{actual} = C_v\, V_{theoretical}}$$

Each orifice has its own coefficient of veloci[illegible]y usually lie in the range (0.97 - 0.99)

<u>The discharge through the orifice</u> = jet area × jet velocity

The area of the jet is the area of the vena contracta not the area of the orifice.

We use a coefficient of contraction to get the area of the jet

$\boldsymbol{A_{actual} = C_c\, A_{orifice}}$

Giving discharge through the orifice:

$\boldsymbol{Q = v\, A}$

$\boldsymbol{Q_{actual} = A_{actual}\, v_{actual}}$

$\boldsymbol{= C_C\, C_v\, A_{orifice}\, v_{theoretical}}$

$\boldsymbol{= C_d\, A_{orifice}\, v_{theoretical}}$

$Q_{actual} = C_d\, A_{orifice} \sqrt{2gh}$ …. (5.17)

Where C_d is the coefficient of discharge ,

$C_d = C_c * C_v$ …. (5.18)

5.7.2. Time wanted to exhaust the water of tank :

We have an expression for the discharge from the tank

$$Q = C_d\, A_o \sqrt{2gh} \;\ldots.(5.18)$$

We can use this to calculate how long it will take for level in the to fall

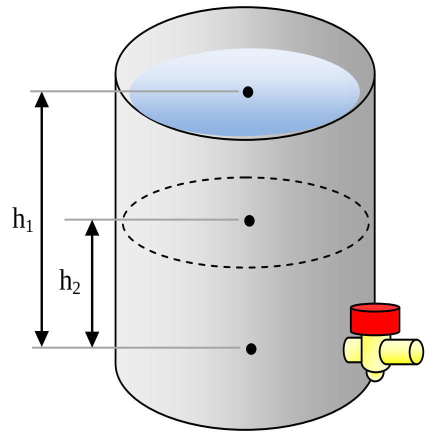
As the tank empties the level of water falls.

The discharge will also drop.

The tank has a cross sectional area of A.

In a time δt the level falls by δh

The flow out of the tank is :



$$Q = v\,A = -A\frac{\delta h}{\delta t} \;\ldots.(5.19)$$ (-ve sign as δh 74 ing)

This Q is the same as the flow out of the orifice so :

$$C_d\, A_o \sqrt{2gh} = -A\frac{\delta h}{\delta t} \;\ldots.(5.20)$$

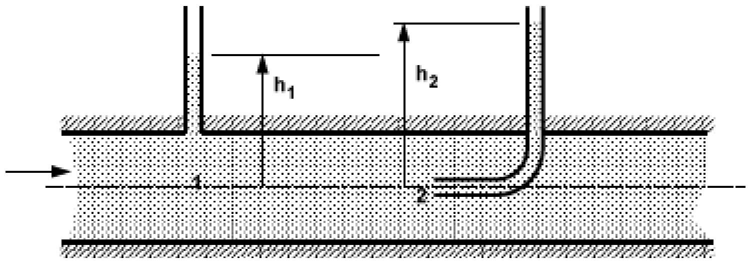
$$\delta t = \frac{-A}{C_d A_o \sqrt{2g}}\,\frac{\delta h}{\sqrt{h}} \;\ldots.(5.21)$$

Integrating between the initial level, h_1, and final level, h_2, gives the time it takes to fall this height :

$$t = \frac{-2A}{C_d A_o \sqrt{2g}}\left[\sqrt{h_2} - \sqrt{h_1}\right] \;\ldots.(5.22)$$

5.7.3. Pitot Static Tube :

Two piezometers, one as normal and one as a Pitot tube within the pipe can be used as shown below to measure velocity of flow.



By applying Bernoulli's eq., we have the equation for p_2 :

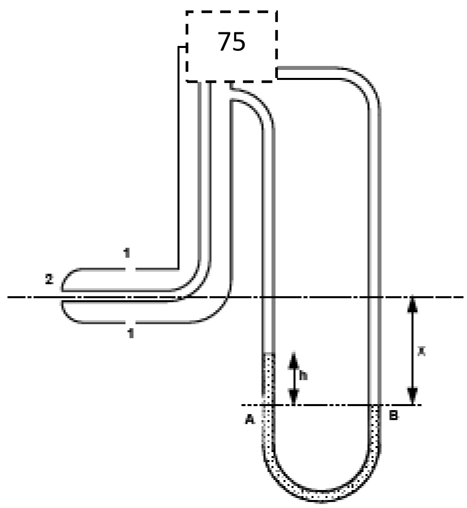
$$P_2 = p_1 + \frac{1}{2}\rho\, v_1^2 \quad \Longrightarrow \quad \rho g h_2 = \rho g h_1 + \frac{1}{2}\rho\, v_1^2$$

$$v = \sqrt{2g\,(h_2 - h_1)} \; \ldots .\, (5.23)$$

We now have an expression for velocity from two pressure measurements and the application of the Bernoulli equation.

The necessity of two piezometers makes this arrangement uncomfortable.

The Pitot static tube combines the tubes and they can then be easily connected to a manometer.



The holes on the side connect to one side of a manometer, while the central hole connects to the other side of the manometer

Using the theory of the manometer,

$P_A = p_1 + \rho g\,(X - h\,) + \rho_{man}\, gh$ *and* $p_B = p_2 + \rho g\, X$

$p_A = p_B \Longrightarrow \quad p_1 + \rho g\,(X - h) + \rho_{man}\, gh = p_2 + \rho g\, X$

we know that $\quad p_2 = p_1 + 0.5\,\rho v_1^2$, giving :

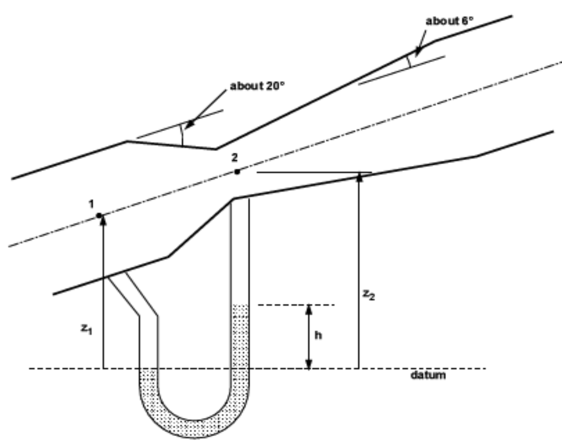
$p_1 + hg\,(\rho_{man} - \rho) = p_1 + 0.5\rho\, v_1^2$

$$v_1 = \sqrt{\frac{2gh(\rho_m - \rho)}{\rho}} \quad \ldots .\,(5.24)$$

The Pitot/Pitot-static is:

- Simple to use (and analyse)
- Gives velocities (not discharge)
- May block easily as the holes are small.

5.7.4. Venturi Meter : The Venturi meter is a device for measuring discharge in a pipe. It is a rapidly converging section which increases the velocity of flow and hence reduces the pressure. It then returns to the original dimensions of the pipe by a gently diverging 'diffuser' section.



Apply Bernoulli along the streamline from point 1 to point 2 (eq. 5.11):

$$\frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{{v_2}^2}{2g} + z_2$$

By continuity equation :

$Q = v_1 A_1 = v_2 A_2 \Longrightarrow v_2 = \frac{v_1 A_1}{A_2}$

Substituting and rearranging gives :

$$\frac{p_1 - p_2}{\rho g} + z_1 - z_2 = \frac{{v_1}^2}{2g}\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right] = \frac{{v_1}^2}{2g}\left[\frac{{A_1}^2 - {A_2}^2}{{A_2}^2}\right] \dots (5.25)$$

$$\therefore v_1 = A_2 \sqrt{\frac{2g[\frac{p_1 - p_2}{\rho g} + z_1 - z_2]}{{A_1}^2 - {A_2}^2}} \dots (5.26)$$

Note : The theoretical (ideal) discharge is v*A and actual discharge takes into account the losses due to friction. We include a coefficient of discharge (Cd ≈ 0.9)

$Q_{ideal} = v_1 A_1$

$Q_{actual} = C_d\, Q_{ideal} = C_d\, v_1 A_1$

$$\therefore Q_{actual} = C_d A_1 A_2 \sqrt{\frac{2g[\frac{p_1 - p_2}{\rho g} + z_1 - z_2]}{{A_1}^2 - {A_2}^2}} \dots (5.27)$$

In terms of the manometer readings :

$p_1 + \rho g z_1 = p_2 + \rho_{man} gh + \rho g (z_2 - h)$

$$\frac{p_1 - p_2}{\rho g} + z_1 - z_2 = h\left(\frac{\rho_{man}}{\rho} - 1\right)$$

Giving

$$\therefore Q_{actual} = C_d A_1 A_2 \sqrt{\frac{2gh[\frac{\rho_{man}}{\rho} - 1]}{{A_1}^2 - {A_2}^2}} \dots (5.28)$$

This expression does not include any elevation terms. (z_1 or z_2)

When used with a manometer, the Venturimeter can be used without knowing its angle.

Venturimeter design:

- The diffuser assures a gradual and steady deceleration after the throat. So that pressure rises to something near that before the meter.
- The angle of the diffuser is usually between 6 and 8 degrees.

- Wider and the flow might separate from the walls increasing energy loss.
- If the angle is less the meter becomes very long and pressure losses again become significant.
- The efficiency of the diffuser of increasing pressure back to the original is rarely greater than 80%.
- Care must be taken when connecting the manometer so that no burrs are present.

5.7.5.Notches and Weirs :

- A notch is an opening in the side of a tank or reservoir.
- It is a device for measuring discharge
- A weir is a notch on a larger scale - usually found in rivers.
- It is used as both a discharge measuring device and a device to raise water levels.
- There are many different designs of weir.
- We will look at sharp crested weirs.

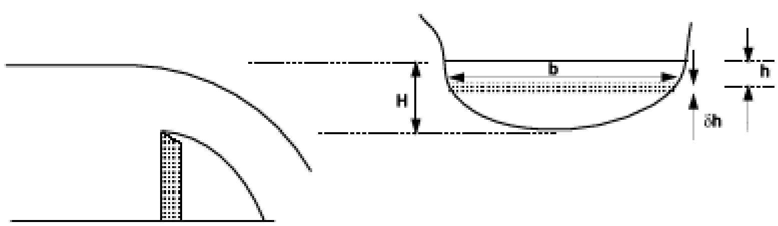
5.7.5.1. Weir Assumptions :

- velocity of the fluid approaching the weir is small so we can ignore kinetic energy.
- The velocity in the flow depends only on the depth below the free surface $\boldsymbol{v = \sqrt{2gh}}$ **.**

These assumptions are fine for tanks with notches or reservoirs with weirs, in rivers with high velocity approaching the weir is substantial the kinetic energy must be taken into account.

5.7.5.2. General Weir Equation :

Consider a horizontal strip of width b, depth h below the free surface



velocity through the strip, $v = \sqrt{2gh}$ and discharge through the strip,

$$\delta Q = Av = b\,\delta h\sqrt{2gh}$$

Integrating from the free surface, h = 0, to the weir crest . $H = H$, gives the total theoretical discharge

$$Q_{theoretical} = \sqrt{2g}\int_0^H bh^{1/2}\,dh \;.... (5.29)$$

This is different for every differently shaped weir or notch. We need an expression relating the width of flow across the weir to the depth below the free surface

5.7.5.3. Rectangular Weir :

The width does not change with depth so **(b = constant = B)** and substituting this into the general weir equation gives :

$$Q_{theoretical} = B\sqrt{2g}\int_0^H bh^{1/2}\,dh = \frac{2}{3}B\sqrt{2g}H^{3/2} \;.... (5.30)$$

To get the actual discharge we introduce a coefficient of discharge, C_d, to account for losses at the edges of the weir and contractions in the area of flow :

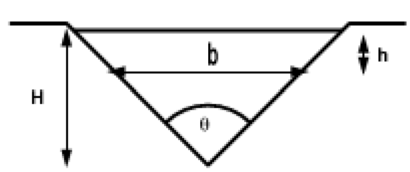
$$Q_{actual} = C_d\frac{2}{3}B\sqrt{2g}H^{3/2} \;.... (5.31)$$

Example(6) (Rectangular Weir) : Water enters the Millwood flood storage area via a rectangular weir when the river height exceeds the weir c For design purposes a flow rate of 162 liters / s over the weir can be assumed

1. ***Assuming a height over the crest of 20cm and Cd=0.2, what is the necessary width, B, of the weir?***
2. ***What will be the velocity over the weir at this design?***

5.7.5.4. 'V' Notch Weir :

The relationship between width and depth is dependent on the angle θ of the "V".



The width, b, a depth h from the free surface is :

$B = 2\,(H - h)\tan\left(\frac{\theta}{2}\right)$

So the discharge is :

$$Q_{theoretical} = 2\sqrt{2g}\tan\left(\frac{\theta}{2}\right)\int_0^H (H-h)h^{1/2}dh = 2\sqrt{2g}\tan\left(\frac{\theta}{2}\right)\left[\frac{2}{3}Hh^{3/2} - \frac{2}{5}h^{5/2}\right]_0^H$$

$$Q_{theoretical} = \frac{8}{15}\sqrt{2g}\tan\left(\frac{\theta}{2}\right)H^{5/2} \quad \dots (5.32)$$

The actual discharge is obtained by introducing a coefficient of discharge :

$$Q_{actual} = C_d\,\frac{8}{15}\sqrt{2g}\tan\left(\frac{\theta}{2}\right)H^{5/2} \quad \dots (5.33)$$

***Example(7) 'V' Notch Weir** Water is flowing over a 90o 'V' Notch weir into a tank with a cross-sectional area of 0.6m2. After 30s the depth of the water in the tank is 1.5m. If the discharge coefficient for the weir is 0.8, what is the height of the water above the weir?*

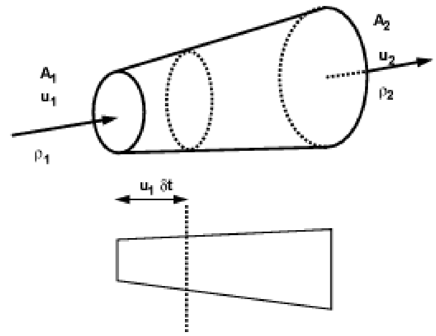
5.8. The Momentum Equation :

is a statement of Newton's Second Law. It relates the sum of the forces to the acceleration or rate of change of momentum. From solid mechanics you will recognize

$$F = ma$$

What mass of moving fluid we should use?

We use a different form of the equation. Consider a stream tube and assume steady non-uniform flow:



In time δt a volume of the fluid moves from the inlet a distance $u_1\delta t$, so

volume entering the stream tube = area × distance =$A_1v_1\delta t$

mass entering stream tube = volume density = $\rho A_1 v_1 \delta t$

momentum entering stream tube = mass velocity = $\rho A_1 v_1 \delta t\, v_1$

Similarly, at the exit, we get the expression:

momentum leaving stream tube = $\rho A_2 u_2 \delta t\, u_2$

By another reading of Newton's 2nd Law.

where Momentum = m * v

Force = mass x acceleration $= m\dfrac{dv}{dt} = \dfrac{dmv}{dt} =$ rate of change of momentum

$$F = \frac{(\rho_2 A_2 v_2\, \delta t\, v_2 - \rho_1 A_1 v_1 \delta t v_1)}{\delta t}$$

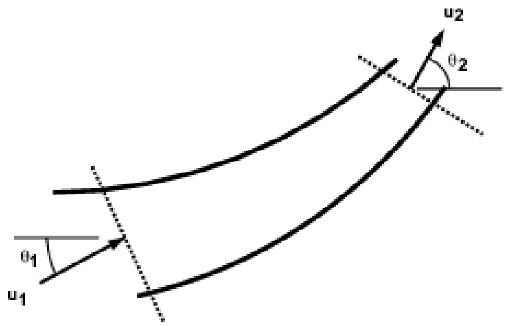
We know from continuity that: $Q = A_1 v_1 = A_2 v_2$

And if we have a fluid of constant density,

$F = Q\rho\,(v_2 - v_1)$ .... (5.34)

The Momentum equation

This force acts on the fluid in the direction of the flow of the fluid



The previous analysis assumed the inlet and outlet velocities in the same direction (i.e. a one dimensional system).

What happens when this is not the case?

We consider the forces by resolving in the directions of the co-ordinate axes. The force in the x-direction :

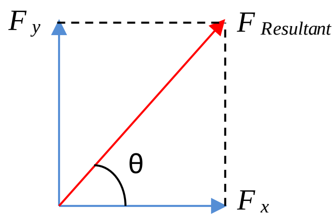
$$F_x = \rho Q\,(v_2 \cos\theta_2 - v_1 \cos\theta_1) = \rho Q\,(v_{2x} - v_{1x})$$

And the force in the y-direction:

$$F_y = \rho Q\,(v_2 \sin\theta_2 - v_1 \sin\theta_1) = \rho Q\,(v_{2y} - v_{1y})$$

The resultant force can be found by combining these components

$$F_{Resultant} = \sqrt{F_x{}^2 + F_y{}^2} \quad \ldots.(5.35)$$



And the angle of this force :

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) \quad \ldots.(5.36)$$

This hydrodynamic force is made up of three components:

F_R = Force exerted on the fluid by any solid body touching the control volume

F_B = Force exerted on the fluid body (e.g. gravity)

F_P = Force exerted on the fluid by fluid pressure outside the control volume

So we say that the total force, ***F_T***, is given by the sum of these forces:

$$F_T = F_R + F_B + F_P$$

The force exerted by the fluid on the solid body touching the control volume is opposite to F_R ***(action force)***.

So the reaction force, ***R***, is given by

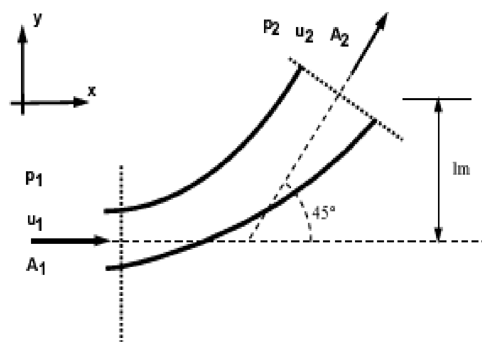
$$\boxed{R = -F_R}$$

5.8.1.Application of the Momentum Equation:

5.8.1.1. Forces on a Bend :

Consider a converging or diverging pipe bend lying in the vertical or horizontal plane turning through an angle of θ.

Here is a diagram of a diverging pipe bend



Why do we want to know the forces here?

As the fluid changes direction a force will act on the bend.

This force can be very large in the case of water supply pipes. The bend must be held in place to prevent breakage at the joints.

We need to know how much force a support (thrust block) must withstand.

Step in Analysis:

1.Draw a control volume

2.Decide on co-ordinate axis system

3.Calculate the total force

4.Calculate the pressure force

5.Calculate the body force

6.Calculate the resultant force.

***Example (8): (Forces on a Bend)** The outlet pipe from a pump is a bend of 45o rising in the vertical plane (i.e. and internal angle of 135o). The bend is 150mm diameter at its inlet and 300mm diameter at its outlet. The pipe axis at the inlet is horizontal and at the outlet it is 1m higher. By neglecting friction, calculate the force and its direction if the inlet pressure is 100kN/m2 and the flow of water through the pipe is 0.3m3/s. The volume of the pipe is 0.075m3. [Ans. 13.95kN at 67o 39' to the horizontal]*

Solution:

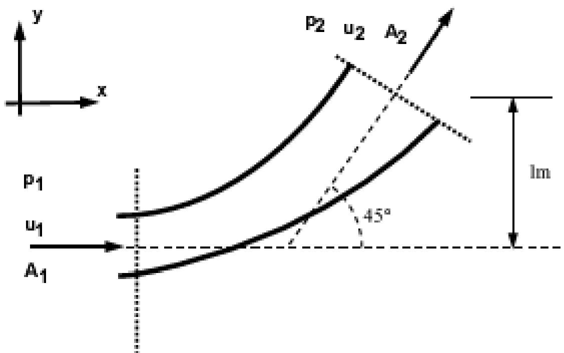
1&2 Draw the control volume and the axis

System

$p_1 = 100\ kN/m^2$,
$Q = 0.3\ m^3/s$
$\theta = 45°$

$d_1 = 0.15\ m$ $d_2 = 0.3\ m$

$A_1 = 0.177\ m^2$ $A_2 = 0.0707\ m^2$



3. <u>Calculate the total force</u>

in the x direction

$$F_{T_x} = \rho Q\left(u_{2_x} - u_{1_x}\right)$$
$$= \rho Q\left(u_2 \cos\theta - u_1\right)$$

by continuity $A_1 u_1 = A_2 u_2 = Q$, so

$$u_1 = \frac{0.3}{\pi\left(0.15^2/4\right)} = 16.98\, m/s$$
$$u_2 = \frac{0.3}{0.0707} = 4.24\, m/s$$

$$F_{T_x} = 1000 \times 0.3(4.24\cos 45 - 16.98)$$
$$= -4193.68\ N$$

and in the y-direction

$$
\begin{aligned}
F_{T_y} &= \rho Q\left(u_{2_y} - u_{1_y}\right) \\
&= \rho Q\left(u_2 \sin\theta - 0\right) \\
&= 1000 \times 0.3(4.24 \sin 45) \\
&= 899.44\ N
\end{aligned}
$$

4. <u>Calculate the pressure force.</u>

$$F_p = \text{pressure force at 1 - pressure force at 2}$$

$$\theta_1 = 0, \qquad \theta_2 = \theta$$

$$F_{p_x} = p_1 A_1 \cos 0 - p_2 A_2 \cos\theta = p_1 A_1 - p_2 A_2 \cos\theta$$

$$F_{p_y} = p_1 A_1 \sin 0 - p_2 A_2 \sin\theta = -p_2 A_2 \sin\theta$$

We know pressure at the inlet, but not at the outlet, we can use the Bernoulli equation to calculate this unknown pressure.

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

The height of the pipe at the outlet is 1m above the inlet.

Taking the inlet level as the datum:

$z_1 = 0$, $z_2 = 1\text{m}$, So the Bernoulli equation becomes:

$$\frac{100000}{1000\times 9.81}+\frac{16.98^2}{2\times 9.81}+0=\frac{p_2}{1000\times 9.81}+\frac{4.24^2}{2\times 9.81}+1.0$$
$$p_2=225361.4\ N/m^2$$

$$F_{P_x}=100000\times 0.0177-225361.4\cos 45\times 0.0707$$
$$=1770-11266.34=-9496.37\ kN$$

$$F_{P_y}=-225361.4\sin 45\times 0.0707$$
$$=-11266.37$$

5. <u>Calculate the body force</u>

The body force is the force due to gravity. That

is the weight acting in the -ve y direction.

$$F_{B_y}=-\rho g\times volume$$
$$=-1000\times 9.81\times 0.075$$

$$F_{By}=-735.75N$$

There are no body forces in the x direction,

$$F_{B_x}=0$$

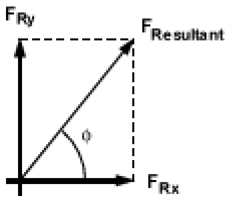
6. <u>Calculate the resultant force</u>

$$F_{T_x}=F_{R_x}+F_{P_x}+F_{B_x}$$
$$F_{T_y}=F_{R_y}+F_{P_y}+F_{B_y}$$

$$F_{R_x}=F_{T_x}-F_{P_x}-F_{B_x}$$
$$=-4193.6+9496.37$$
$$=5302.7\ N$$

$$F_{R_y}=F_{T_y}-F_{P_y}-F_{B_y}$$
$$=899.44+11266.37+735.75$$
$$=12901.56\ N$$

And the resultant force by the fluid is given by :



$$F_R = \sqrt{F_{R\,x}^2 - F_{R\,y}^2}$$
$$= \sqrt{5302.7^2 + 12901.56^2}$$
$$= 13.95\,kN$$

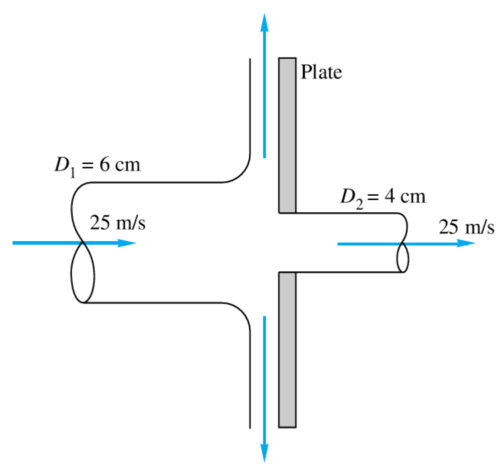
And the direction of application is

$$\phi = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right)$$
$$= \tan^{-1}\left(\frac{12901.56}{5302.7}\right)$$
$$= 67.66^\circ = 67^\circ\,39'$$

The reaction force by the bend is the same magnitude but in the opposite direction

$$R = -F_R = -13.95\,kN$$

Example (9): ***The 6-cm-diameter 20°C water jet in Fig. strikes a plate containing a hole of 4-cm diameter. Part of the jet passes through the hole, and part is deflected. Determine the horizontal force required to hold the plat***



Solution :

$$Q_{in} = \frac{\pi}{4}(0.06)^2 25 = 0.0707\, m^3/s$$

$$Q_{hole} = \frac{\pi}{4}(0.04)^2 25 = 0.0314\, m^3/s$$

$$F = \rho Q(V_{out} - V_{in})$$

for devided or branched flow

$$F_x = \rho(\sum Q_{out} V_{outx} - \sum Q_{in} V_{inx})$$

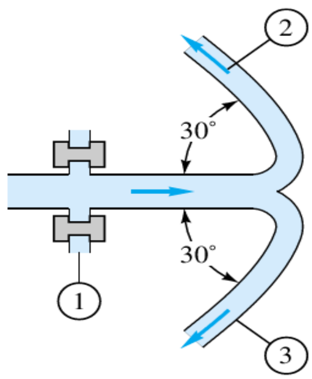
$$F_x = 998(0 + 0 + 0.0314 \times 25 - 0.0707 \times 25)$$

$$F_x = -980\ N$$

$$R_x = 980 N$$

$$F_y = 0$$

Example (10): **Water at 20°C exits to the standard sea-level atmosphere through the split nozzle in Fig. Duct areas are A1 = 0.02 m2 and A2 = A3 = 0.008 m2. If p1= 135 kPa (absolute) and the flow rate is Q2 = Q3 = 275 m3/h, compute the force on the flange bolts at section 1.**



Solution: With the known flow rates, we can compute the various velocities:

$$V_2 = V_3 = \frac{275/3600\ \mathrm{m^3/s}}{0.008\ \mathrm{m^2}} = 9.55\ \frac{\mathrm{m}}{\mathrm{s}}; \quad V_1 = \frac{550/3600}{0.02} = 7.64\ \frac{\mathrm{m}}{\mathrm{s}}$$

The CV encloses the split nozzle and cuts through the flange. The balance of forces is

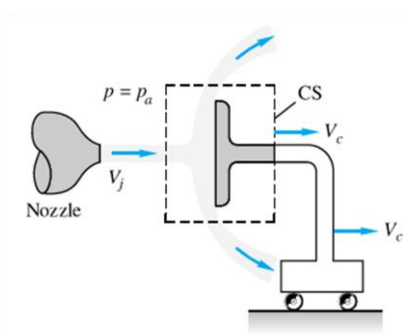
$$\sum F_x = -F_{bolts} + p_{1,gage}A_1 = \rho Q_2(-V_2\cos 30°) + \rho Q_3(-V_3\cos 30°) - \rho Q_1(+V_1),$$

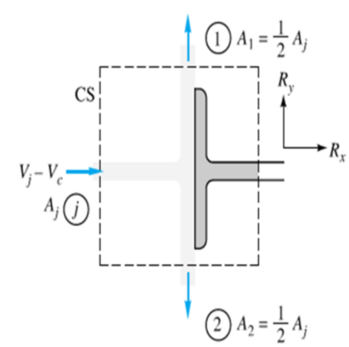
$$\text{or:} \quad F_{bolts} = 2(998)\left(\frac{275}{3600}\right)(9.55\cos 30°) + 998\left(\frac{550}{3600}\right)(7.64) + (135000 - 101350)(0.02)$$

$$= 1261 + 1165 + 673 \approx \mathbf{3100\ N} \quad \textit{Ans.}$$

Example (11): ***A water jet of velocity V_j impinges normal to a flat plate which moves to the right at velocity V_c, as shown in Fig. Find the force required to keep the plate moving at constant velocity if the jet density is 1000 kg/m3, the jet area is 3 cm^2, and Vj and Vc are 20 and 15 m/s, respectively. Neglect the weight of the jet and plate, and assume steady flow with respect to the moving plate with the jet splitting into an equal upward and downward half-jet.***

Solution:





For moving control volume with V=Vc we have

$$V_{in} = V_j - V_c = 20 - 15 = 5\,m/s$$

By continuity equation we have:

$$Q_{in} = Q_{out}$$

$$A_j V_{in} = A_1 V_1 + A_2 V_2 \,, \; A_1 = A_2 = \frac{1}{2} A_j$$

$$V_{in} = \frac{1}{2} V_1 + \frac{1}{2} V_2 \,, \quad but \; from\, symmetry\, and\, neglacting\, the\, wight : V_1 = V_2$$

$$V_{in} = V_1 = V_2$$

$$F = \rho Q(V_{out} - V_{in})$$

for devided or branched flow

$$F_x = \rho(\sum Q_{out} V_{outx} - \sum Q_{in} V_{inx})$$

$$F_x = \rho(\sum A_{out} V_{out} V_{outx} - \sum A_{in} V_{in} V_{inx})$$

$$F_x = \rho(0 - 1000 \times 0.0003 \times 5 \times 5)$$

$$F_x = -7.5\ N$$

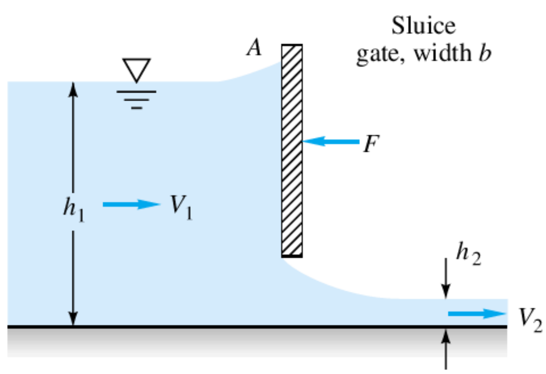
$$F_x = F_{px} + F_{Rx}$$

$$F_{Rx} = F_x - F_{px} = -7.5 - 0 = -7.5$$

$$R_x = -7.5N$$

$$F_y = 0$$

Example (12): ***The sluice gate in Fig. can control and measure flow in open channels. At sections 1 and 2, the flow is uniform and the pressure is hydrostatic. The channel width is b into the paper. Neglecting bottom friction, derive an expression for the force F required to hold the gate. For what condition h2 is the force largest? For very low velocity $V_1^2 << gh1$, for what value of h2/h1 will the force be one-half of the maximum?***



Solution: The CV encloses the inlet and exit and the whole gate, as shown. From mass conservation, the velocities are related by

$$V_1h_1b = V_2h_2b, \quad \text{or:} \quad V_2 = V_1(h_1/h_2)$$

The bottom pressures at sections 1&2 equal ρgh_1 and ρgh_2, respectively. The horizontal force balance is

$$\sum F_x = -F_{gate} + \frac{1}{2}\rho gh_1(h_1b) - \frac{1}{2}\rho gh_2(h_2b) = \dot{m}(V_2 - V_1), \quad \dot{m} = \rho h_1bV_1$$

$$\text{Solve for} \quad \mathbf{F_{gate} = \frac{1}{2}\rho gbh_1^2[1-(h_2/h_1)^2] - \rho h_1bV_1^2\left[\frac{h_1}{h_2}-1\right]} \quad \textit{Ans.}$$

For everything held constant except h_2, the maximum force occurs when

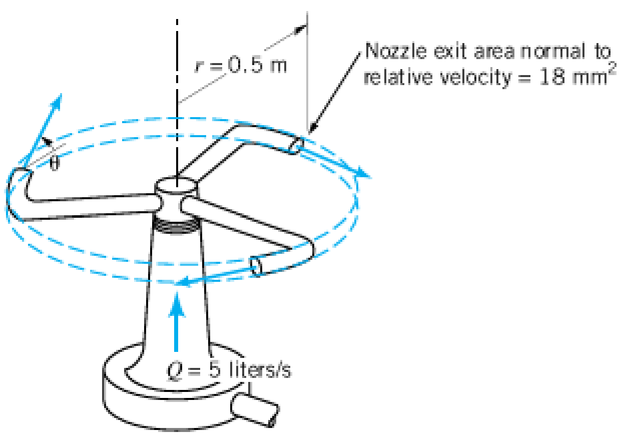
$$\frac{dF}{dh_2} = 0 \quad \text{which yields} \quad h_2 = \left(V_1^2h_1^2/g\right)^{1/3} \quad \text{or:} \quad \frac{h_2}{h_1} \approx \left(\frac{\mathbf{V_1^2}}{\mathbf{gh_1}}\right)^{1/3} \quad \textit{Ans.}$$

Finally, for very low velocity, only the first term holds: $F \approx (1/2)\rho gb\left(h_1^2 - h_2^2\right)$. In this case the maximum force occurs when $h_2 = 0$, or $F_{max} = (1/2)\rho gbh_1^2$. (Clearly this is the

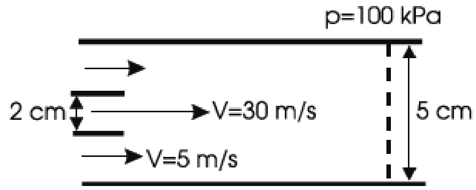
special case of the earlier results for F_{max} when $V_1 = 0$.) Then for this latter case of very low velocity,

$$F = (1/2)F_{max} \quad \text{when} \quad \mathbf{h_2 = h_1/\sqrt{2}}. \quad \textit{Ans.}$$

Example (13) : **Five liters/s of water enter the rotor shown in Fig. along the axis of rotation. The cross-sectional area of each of the three nozzle exits normal to the relative velocity is 18 mm2. How large is the resisting torque required to hold the rotor stationary? How fast will the rotor spin steadily if the resisting torque is reduced to zero and (a) $\theta = 0°$, (b) $\theta = 30°$, (c) $\theta = 60°$?**

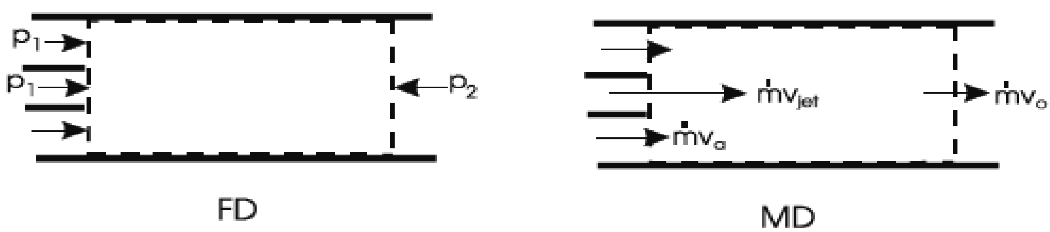


Example (14) : ***In the eductor shown in the following figure, water is injected through a 2-cm nozzle at a speed of 30 m/s. The flow of water in the 5-cm duct is 5 m/s. If the pressure downstream where the flow is totally mixed is 100 kPa, what is the pressure where the water is injected through the nozzle? Neglect the friction on the walls.***



Solution

Draw a control surface as shown with the appropriate force and momentum diagrams.



The velocity at the outlet may be obtained from the continuity equation for a steady flow. The mass flow in the high-speed jet is

$$\begin{aligned}\dot{m}_{jet} &= 1000 \times \frac{\pi}{4} \times 30 \times 0.02^2 \\ &= 9.42 \text{ kg/s}\end{aligned}$$

The mass flow through the outer annular region is

$$\begin{aligned}\dot{m}_a &= 1000 \times \frac{\pi}{4} \times 5 \times (0.05^2 - 0.02^2) \\ &= 8.24 \text{ kg/s}\end{aligned}$$

The velocity at the outlet is

$$\begin{aligned}V_o &= (\dot{m}_{jet} + \dot{m}_a)/\rho A \\ &= (9.42 + 8.24)/(1000 \times \frac{\pi}{4} \times 0.05^2) \\ &= 9 \text{ m/s}\end{aligned}$$

The sum of the forces on the control surface is

$$\begin{aligned}\sum_{cs} F_x &= p_1 A - p_2 A \\ &= \frac{\pi}{4} \times 0.05^2 \times (p_1 - 10^5) \\ &= 0.00196 \times (p_1 - 10^5)\ \text{N}\end{aligned}$$

The control surface is not accelerating and the flow is steady, so the momentum flux is

$$\begin{aligned}\sum_{cs} \dot{m}_o v_{o,x} - \sum_{cs} \dot{m}_i v_{i,x} &= (9.42 + 8.24) \times 9 - 9.42 \times 30 - 8.24 \times 5 \\ &= -164.86\ \text{kg} \cdot \text{m/s}^2\end{aligned}$$

Equating the force and momentum flux

$$\begin{aligned}0.00196 \times (p_1 - 10^5) &= -164.86 \\ &= \underline{\underline{15.9\ \text{kPa}}}\end{aligned}$$

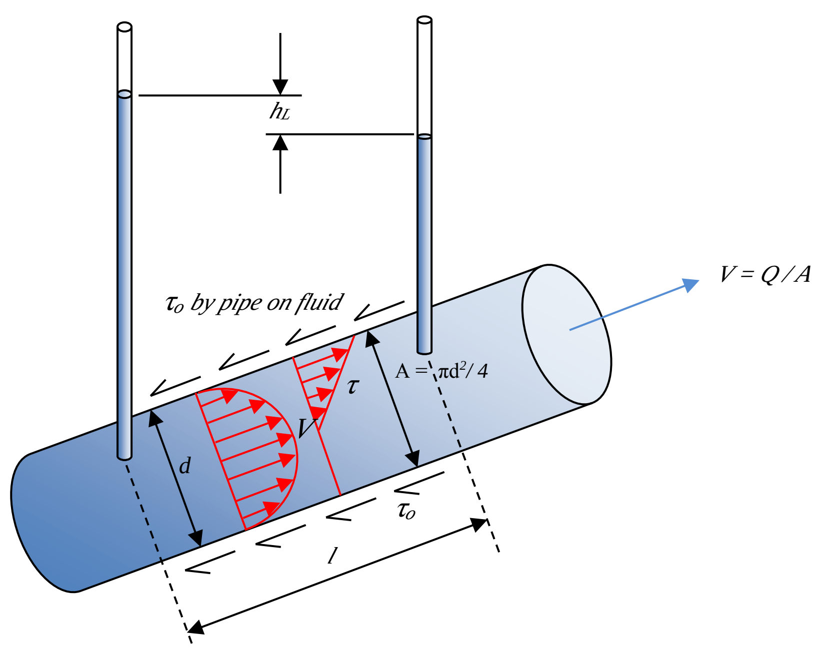
Chapter Six

REAL FLOW IN CLOSED PIPES

6.1. Introduction :

The flow of a real fluid is vastly more complex than that of an ideal fluid, owing to phenomena caused by the existence of viscosity. Viscosity introduces resistance to motion by causing shear or friction forces between fluid particles and between these and boundary walls. For flow to take place, work must be done against these resistance forces, and in the process energy is converted into heat.

6.2. Fundamental Equations :



6.2.1. Reynolds Number :

The effect of viscosity cause the flow of a real fluid to occur under three very different conditions :

1. Laminar flow
2. Transitional flow
3. Turbulent flow

The characteristics of these regimes demonstrated by Reynolds number (R):

$R = \text{Reynolds number} = \frac{\rho V D}{\mu} = \frac{V\,d}{\nu} \ldots . (6.1)$ (dimensionless value)

In which :

V = the mean velocity of the fluid in the pipe (m/s)

d = diameter of the pipe (m)

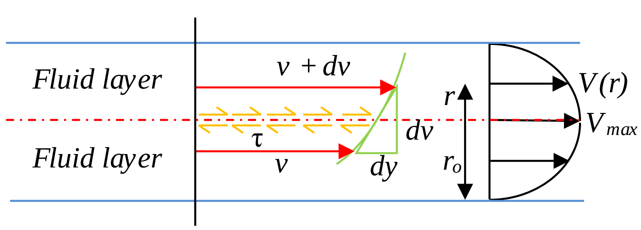
ρ = mass density of the fluid (kg/m^3)

μ = dynamic viscosity ($N.\ s/m^2$ or Pa .s or kg/m .s)

ν = kinematic viscosity (m^2/s)

Reynolds number for various flow regimes	
Flow Regime	**Reynolds Number**
Laminar	< 2000
Transitional	2000-4000
Turbulent	> 4000

1. ***In laminar flow***, agitation of fluid particles is of molecular nature only, and these particles are constrained to motion in essentially parallel paths by the action of viscosity. The shearing stress between adjacent moving layers is determined in laminar flow by the viscosity and is completely defined by the differential equation which was discussed in chapter one :



laminar flow occurs when Re ≤ 2000. Laminar flow in a round tube is called *Poiseuille* flow or *Hagen-Poiseuille* flow in honor of pioneering researchers who studied low-speed flows in the 1840s. For **laminar flow only** in pipes, shear stresses through flow is related by Newton law of viscosity

$$\tau = \mu \frac{dv}{dy}$$

where y is the distance from the pipe wall. Change variables by letting $y = r_0 - r$, where r_0 is pipe radius and r is the radial coordinate. Next, use the chain rule of- calculus :

$$\tau = \mu \frac{dv}{dy} = \mu \left(\frac{dv}{dr}\right)\left(\frac{dr}{dy}\right) \quad \dots . (6.2)$$

Rearrangement and substitutions with r =D/2 :

$$h_L = \Delta h = \frac{\tau_w 2\, L}{r\gamma} = -\mu \left(\frac{dv}{dr}\right) 2 \left(\frac{L}{r\gamma}\right) \quad \dots . (6.3)$$

$$dv = -\frac{h_L\, \gamma}{2\mu\, L}\, r\, dr \dots . (6.4)$$

By integral with boundary condition of $r = 0$, $V = V_{max}$

$$V = V_{max} - \frac{h_L\, \gamma}{2\mu\, L}\, r^2 \quad \dots . (6.5)$$ (parabolic velocity profile)

From this equation we find that (at $r=r_o=D/2$, $V=0$):

$$V_{max} = \frac{h_L\, \gamma}{16\mu\, L}\, D^2 \quad \dots . (6.6)$$

The mean velocity= $V_{mean} = Q/A$ …. (6.7)

We find discharge by integration of *V.dA* overall the section area and we arrived to following result :

$$V_{mean} = \frac{1}{2} V_{max} = \frac{h_L \gamma}{32\, \mu\, L}\, D^2 \quad \dots . (6.8)$$

$$h_L = 32\, \frac{\mu\ L}{\gamma D^2}\, V \dots . (6.9)$$ (Hagen-Poisseuille equation, 1839,40)

By comparing this equation with Darcy equation, we find that:

$$f = \frac{64}{R} \ \ldots.(6.10)$$

For horizontal pipes:

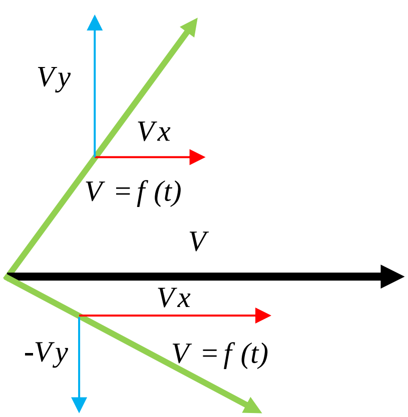
$$\Delta P = (64/Re)(\rho V^2/2)(L/D) = (64\mu/\rho Vd)(\rho V^2/2)(L/D) = 32\mu VL/d^2 \ \ldots.(6.11)$$

- Sometimes it's more convenient to deal with volume flow rate (Q) rather than velocity (V). Thus we can write one last relation:

$$\Delta P = (128/\pi)\mu QL/D^4 \ \ldots.(6.12)$$ (laminar flow only in horizontal pipes)

Note the significance of this result: if you double the flow rate **Q** or the length of the pipe **L**, the pressure drop doubles (makes sense.) Also, for a given flow rate Q, if you double the diameter of the tube, the pressure drop decreases by a factor of 16! So use a little bigger pipe in your plumbing design.

2. ***In turbulent flow***, fluid particles do not remain in layers, but move in a heterogeneous fashion through the flow, sliding past other particles and colliding with some in an entirely haphazard manner that results in a rapid and continuous mixing of the fluid as flow occurs. These particles are observed to travel in randomly moving fluid masses of varying sizes called eddies.



the friction factor depends not only on **Re** but also the roughness of the pipe wall, which is characterized by a ***roughness factor* = e/d**, where **ε** is a measure of the roughness (i.e. height of the bumps on the wall) and **d** is (as always) the pipe diameter. The combined effects of roughness and Re are presented in terms of the *Moody chart* (R,e/D) still needs to be determined. For laminar flow, there is an exact solution for *f* since laminar pipe flow has an exact solution. For turbulent flow, approximate solution for *f* using log-law as per Moody diagram and discussed late or other approximations.

6.2.2. Darcy – Weisbach Equation

$$h_L = f\ \frac{l}{2g}\ \frac{v^2}{D} \ldots .(6.13)$$

In which :

h_L = head loss through a pipe due to friction effects (m)

f = friction factor

l = pipe length (m)

g = gravitational acceleration (m / s^2)

D = pipe diameter (m)

v = fluid velocity (m/s)

this equation, usually called the "Darcy equation'' is still the basic equation for head loss caused by established pipe friction (not pipe fittings) in long, straight, uniform pipes.

The equation of frictional stress is :

$$\tau_o = \frac{f\rho V^2}{8} \ldots .(6.14)$$

$\tau_o = F\ (v, d, \rho, \mu, e)$

And the velocity of friction may be expressed by :

$$V_o = \sqrt{\frac{\tau_o}{\rho}} = V\ \sqrt{\frac{f}{8}} \ldots .(6.15)$$

6.3. Equations which relate between Reynolds number and friction factor (in case of turbulent flow) :

1. Blasius Equation :

$$\mathrm{f} = \frac{0.316}{\mathrm{R}^{0.25}} \qquad \mathrm{R}\ \leq\ 10^5 \ldots .(6.16)$$

1. Burka Equation :

$$f = \frac{0.21}{R^{0.21}} \quad \dots (6.17)$$

2. Von Karman - Prandtl Equations :

$$\frac{1}{\sqrt{f}} = 2 \log \left(\frac{R\sqrt{f}}{2.51}\right) \quad \dots (6.18)$$

$$\frac{1}{\sqrt{f}} = 2 \log \left(\frac{3.7\,D}{e}\right) \quad \dots (6.19)$$

3. Barr Equation :

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{e}{3.7\,D} + \frac{5.1286}{R^{0.89}}\right) \quad \dots (6.20)$$

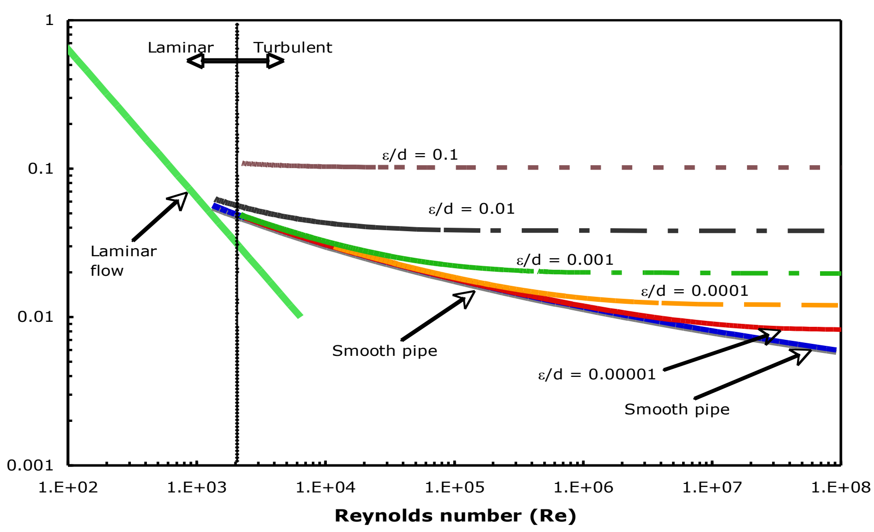
4. Kolebrook-White Equation

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{e}{3.7\,D} + \frac{2.51}{R\sqrt{f}}\right) \quad \dots (6.21)$$

5. Swamee Equation

$$f = \frac{0.25}{\left[\log\left(\frac{e}{3.7\,D} + \frac{5.74}{R^{0.9}}\right)\right]^2} \quad \dots (6.22)$$

$3*10^3 \leq R \leq 3*10^8$ & $10^{-6} \leq \frac{e}{D} \leq 2*10^{-2}$



- Pipe Flow Problem Types

Variable	Type I	Type II	Type III
a. Fluid **Density**	Given	Given	Given

Viscosity	Given	Given	Given
b. Pipe			
Diameter	Given	Given	**Determine**
Length	Given	Given	Given
Roughness	Given	Given	Given
c. Flow			
Flowrate or Average Velocity	Given	**Determine**	Given
d. Pressure			
Pressure Drop or Head Loss	**Determine**	Given	Given

$$\tau = \varepsilon \frac{dv}{dy} \quad \ldots.(6.23)$$

τ = turbulent shear stress

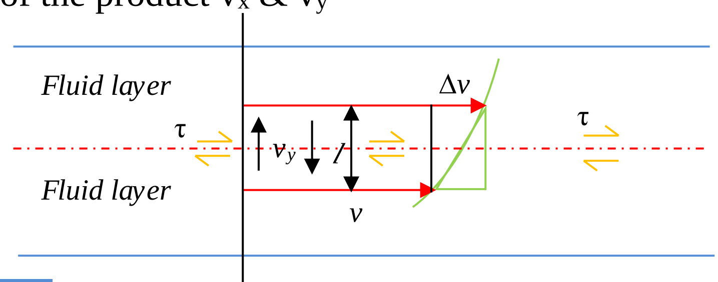
ε = eddy viscosity

$$\tau_R = -\rho \overline{v_x v_y} \quad \ldots.(6.24)$$

τ_R = Reynolds stress

v_x , v_y = velocities of fluid particles in turbulent flow conditions at x and y directions respectively.

$\overline{v_x v_y}$ = mean value of the product v_x & v_y



$$l = -k \frac{(\frac{dv}{dy})}{(\frac{d^2v}{dy^2})} \quad \ldots.(6.25)$$

In which : l = mixing length (mean distance in which the fluid particles transported by turbulence between layers)

k = dimensionless constant of turbulence

Example (1): ***Water of kinematic viscosity 1.15*10^{-6} m^2/s flows in a cylindrical pipe of 30 mm diameter. Calculate the largest flow rate for which laminar flow can be expected. What is the equivalent flow rate for air ?***

Solution : taking R = 2100 as the conservative upper limit for laminar flow,

$$2100 = \frac{V\,(0.03)}{0.00000115} \quad ; \quad V = 0.0805 \ \ mps$$

$$Q_{\text{water}} = 0.0805 * \frac{\pi}{4}\,(0.03)^2 = 5.69 * 10^{-5} \ \text{m}^3/\text{s}$$

$\nu_{air} = 1.37 * 10^{-5}$ m^2/s and $Q_{air} = 6.78*10^{-4}$ m^3/s □ ≈ 12 Q_{water}

Example (2): ***Water flows in a 150 mm diameter pipeline at a mean velocity of 4.5 mps. The head lost in 30 m of this pipe is measured experimentally and found to be 5.33 m. calculate the friction velocity.***

Solution :

$$5.33 = f\,\frac{30}{0.15}\,\frac{4.5^2}{2g} \quad ; \quad f = 0.026$$

$$V_o = 4.5\sqrt{\frac{0.026}{8}} = 0.26 \text{ mps}$$

Example (3): ***A turbulent flow of water occurs in a pipe of 2 ft diameter. The velocity profile is measured experimentally and found to be closely approximated by the equation v = 10 + 0.8 ln y, in which v is in ft/sec and y (the distance from the pipe wall)is in feet. The shearing stress in the fluid at a point 4 in. from the wall is calculated analytically from measurements of pressure drop to be 0.2 psf. Calculate the eddy viscosity, mixing length and turbulence constant at this point.***

Solution :

$$\frac{dv}{dy} = \frac{d}{dy}\,(10 + 0.8\ln y) = \frac{0.8}{y} = \frac{0.8}{(\frac{4}{12})} = 2.4$$

$$\frac{d^2v}{dy^2} = \frac{d}{dy}\left(\frac{0.8}{y}\right) = \frac{-0.8}{y^2} = \frac{-0.8}{(\frac{4}{12})^2} = -7.2$$

$0.2 = \varepsilon\,(2.4)\ ;\ \varepsilon = 0.083$ lb.sec / ft^2

$0.2 = 1.94\, l^2\,(2.4)^2 \ ;\ l = 0.134$ ft

$$0.2 = \frac{1.94\,k^2 (2.4)^4}{(-7.2)^2} \quad ; \quad k = 0.401$$

$$0.2 = 1.94\,k^2\,(\tfrac{1}{3})^2\,(2.4)^2 \quad ; \quad k = 0.401$$

$$l = \frac{-0.401\,(2.4)}{(-7.2)} = 0.134 \ \text{ft}$$

$l = 0.401\left(\frac{4}{12}\right) = 0.134$ ft

The magnitude of the eddy viscosity ε when compared with the viscosity μ (approximately 0.00002lb.sec / ft^2) is of special interest in that it provides a direct comparison between the ***large*** turbulent shear and ***small*** laminar shear f same velocity gradient. The mixing length, l , when compared with the pipe radius is fo be about10 % of the latter dimension; this is a nominal value of correct order of magnitude, as is the turbulence constant, k .

Example (4) : ***(Head loss for laminar flow) Oil (S = 0.85) with a kinematic viscosity of 6 × 10-4 m2/s flows in a 15 cm pipe at a rate of 0.020 m3/s. What is the head loss per 100 m length of pipe?***

Problem Definition

1. Oil is flowing in a pipe at a flow rate of Q = 0.02 m3/s.

2. Pipe diameter is D = 0.15 m.

Find: Head loss (in meters) for a pipe length of 100 m.

Properties: Oil: S = 0.85, ν = 6 × 10-4 m2/s.

Solution :

1. Mean velocity

$$V = \frac{Q}{A} = \frac{0.020\ \mathrm{m^3/s}}{(\pi D^2)/4} = \frac{0.020\ \mathrm{m^3/s}}{\pi((0.15\ \mathrm{m})^2/4)} = 1.13\ \mathrm{m/s}$$

2. Reynolds number

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(1.13\ \mathrm{m/s})(0.15\ \mathrm{m})}{6\times10^{-4}\ \mathrm{m^2/s}} = 283$$

3. Since Re < 2000, the flow is laminar.

4. Head loss (laminar flow).

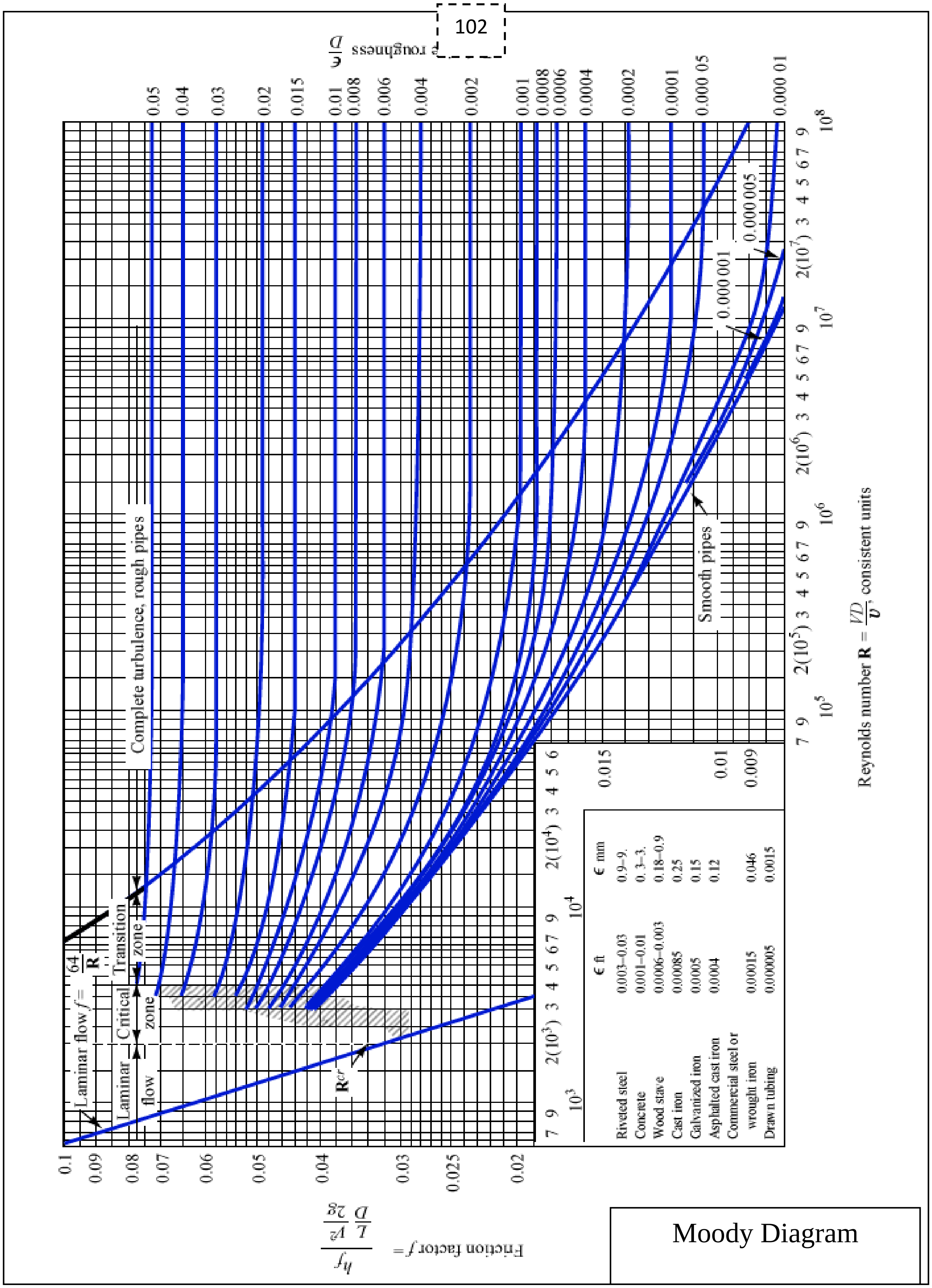
$$h_f = \frac{32\mu LV}{\gamma D^2} = \frac{32\rho\nu LV}{\rho g D^2} = \frac{32\nu LV}{gD^2}$$
$$= \frac{32(6\times10^{-4}\ \mathrm{m^2/s})(100\ \mathrm{m})(1.13\ \mathrm{m/s})}{(9.81\ \mathrm{m/s^2})(0.15\ \mathrm{m})^2} = \boxed{9.83\ \mathrm{m}}$$

Review: Tip! An alternative way to calculate head loss for laminar flow is to use the Darcy-Weisbach equation as follows:

$$f = \frac{64}{\text{Re}} = \frac{64}{283} = 0.226$$

$$h_f = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) = 0.226\left(\frac{100\ \text{m}}{0.15\ \text{m}}\right)\left(\frac{(1.13\ \text{m/s})^2}{2 \times 9.81\ \text{m/s}^2}\right)$$

$$= 9.83\ \text{m}$$



Moody Diagram

***Example (5)*: (Type I) Determine the head lost to friction when water at 15 ∘C, ν = 1.14 mm2 ⋅ s−1 , flows through 300 m of 150 mm diameter galvanized steel pipe (ε = 0.15 mm) at 50 L ⋅ s−1.**

Solution :

$V = 50 \times 10^{-3} m^3 \cdot s^{-1}/((\pi/4)(0.15)^2 m^2) = 2.83\ m\cdot s^{-1}$

$Re = VD/\nu = 2.83\ m\cdot s^{-1} \times 0.15\ m/(1.14 \times 10^{-6}\ m^2 \cdot s^{-1}) = 3.72 \times 10^5$

For galvanized steel, $\varepsilon /D = 0.001$

From Moody Diagram $f = 0.0208$, so

$H_L = 4 \times 0.0208 \times 300\ m/(0.15\ m) \times (2.83\ m\cdot s^{-1})^2/(19.62\ m\cdot s^{-2}) = 67.89\ m$,

say 68 m

***Example (6)*: (Type II) Oil, with ρ =950 kg/m3 and ν = 2 E-5 m2/s, flows through a 30-cm-diameter pipe 100 m long with a head loss of 8 m. The roughness ratio is ε/d = 0.0002. Find the average velocity and flow rate.**

Solution:

By definition, the friction factor is known except for V:

$$f = h_f \frac{d}{L}\frac{2g}{V^2} = (8\text{ m})\left(\frac{0.3\text{ m}}{100\text{ m}}\right)\left[\frac{2(9.81\text{ m/s}^2)}{V^2}\right] \quad \text{or} \quad fV^2 \approx 0.471 \quad \text{(SI units)}$$

To get started, we only need to guess f, compute $V = \sqrt{0.471/f}$, then get Re_d, compute a better f from the Moody chart, and repeat. The process converges fairly rapidly. A good first guess is the "fully rough" value for $\epsilon/d = 0.0002$, or $f \approx 0.014$ from Fig. 6.13. The iteration would be as follows:

Guess $f \approx 0.014$, then $V = \sqrt{0.471/0.014} = 5.80$ m/s and $Re_d = Vd/\nu \approx 87{,}000$. At $Re_d = 87{,}000$ and $\epsilon/d = 0.0002$, compute $f_{new} \approx 0.0195$ [Eq. (6.64)].

New $f \approx 0.0195$, $V = \sqrt{0.481/0.0195} = 4.91$ m/s and $Re_d = Vd/\nu = 73{,}700$. At $Re_d = 73{,}700$ and $\epsilon/d = 0.0002$, compute $f_{new} \approx 0.0201$ [Eq. (6.64)].

Better $f \approx 0.0201$, $V = \sqrt{0.471/0.0201} = 4.84$ m/s and $Re_d \approx 72{,}600$. At $Re_d = 72{,}600$ and $\epsilon/d = 0.0002$, compute $f_{new} \approx 0.0201$ [Eq. (6.64)].

We have converged to three significant figures. Thus our iterative solution is

$$V = 4.84\text{ m/s}$$

$$Q = V\left(\frac{\pi}{4}\right)d^2 = (4.84)\left(\frac{\pi}{4}\right)(0.3)^2 \approx 0.342\text{ m}^3\text{/s} \qquad \textit{Ans.}$$

Example (7) : (Type III) Work previous Example backward, assuming that Q = 0.342 m3/s and ε = 0.06 mm are known but that d (30 cm) is unknown. Recall L = 100 m, ρ = 950 kg/m3, ν = 2 E-5 m2/s, and hL = 8m

Solution:

First write the diameter in terms of the friction factor:

$$f = \frac{\pi^2}{8}\frac{(9.81 \text{ m/s}^2)(8 \text{ m})d^5}{(100 \text{ m})(0.342 \text{ m}^3/\text{s})^2} = 8.28d^5 \qquad \text{or} \qquad d \approx 0.655 f^{1/5} \tag{1}$$

in SI units. Also write the Reynolds number and roughness ratio in terms of the diameter:

$$\text{Re}_d = \frac{4(0.342 \text{ m}^3/\text{s})}{\pi(2 \text{ E-5 m}^2/\text{s})d} = \frac{21{,}800}{d} \tag{2}$$

$$\frac{\epsilon}{d} = \frac{6 \text{ E-5 m}}{d} \tag{3}$$

Guess f, compute d from (1), then compute Re_d from (2) and ϵ/d from (3), and compute a better f from the Moody chart or Eq. (6.64). Repeat until (fairly rapid) convergence. Having no initial estimate for f, the writer guesses $f \approx 0.03$ (about in the middle of the turbulent portion of the Moody chart). The following calculations result:

$$f \approx 0.03 \qquad d \approx 0.655(0.03)^{1/5} \approx 0.325 \text{ m}$$

$$\text{Re}_d \approx \frac{21{,}800}{0.325} \approx 67{,}000 \qquad \frac{\epsilon}{d} \approx 1.85 \text{ E-4}$$

$$f_{\text{new}} \approx 0.0203 \qquad \text{then} \qquad d_{\text{new}} \approx 0.301 \text{ m}$$

$$\text{Re}_{d,\text{new}} \approx 72{,}500 \qquad \frac{\epsilon}{d} \approx 2.0 \text{ E-4}$$

$$f_{\text{better}} \approx 0.0201 \qquad \text{and} \qquad d = 0.300 \text{ m} \qquad \textit{Ans.}$$

Example (8) : ***SAE 30 oil at 20°C flows in the 3-cm-diameter pipe in Fig. which slopes at 37°. For the pressure measurements shown, determine (a) whether the flow is up or down and (b) the flow rate in m3/h***

$$HGL_B = \frac{p_B}{\rho g} + z_B = \frac{180000}{891(9.81)} + 15 = 35.6 \text{ m}; \quad HGL_A = \frac{500000}{891(9.81)} + 0 = 57.2 \text{ m}$$

$$\text{Since} \quad HGL_A > HGL_B \text{ the } \textbf{flow is } \boldsymbol{up} \quad \textit{Ans.} \text{(a)}$$

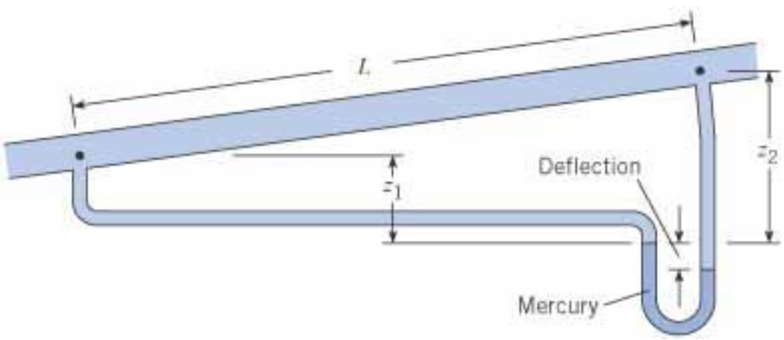
The head loss is the difference between hydraulic grade levels:

$$h_f = 57.2 - 35.6 = 21.6 \text{ m} = \frac{128\mu LQ}{\pi\rho g d^4} = \frac{128(0.29)(25)Q}{\pi(891)(9.81)(0.03)^4}$$

$$\text{Solve for} \quad Q = 0.000518 \text{ m}^3/\text{s} \approx \mathbf{1.86\ m^3/h} \quad \textit{Ans.} \text{ (b)}$$

$$\text{Finally, check} \quad Re = 4\rho Q/(\pi\mu d) \approx 68 \text{ (OK, laminar flow).}$$

Example (9) : ***The velocity of oil (S = 0.8) through the 5 cm smooth pipe is 1.2 m/s. ere L = 12 m, z1 = 1 m, z2 = 2 m, and the manometer deflection is 10 cm. Determine the flow direction, the resistance coefficient f, whether the flow is laminar or turbulent, and the viscosity of the oil.***



Solution:

Based on the deflection on the manometer, the piezometric head on the right side of the pipe is larger than that on the left side. Since the velocity at 1 and 2 is the same, the energy at location 2 is higher than the energy at location 1. Since the a fluid will move from a location of high energy to a location of low energy, the flow is downward (from right to left).

From energy principles:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+h_L$$

$$h_L=\frac{\Delta p}{\gamma}+\Delta z=f\,\frac{L}{D}\frac{V^2}{2g}$$

Manometer equation

$$p_2+(2\,\mathrm{m})\,\gamma_{\mathrm{oil}}+(0.1\,\mathrm{m})\,\gamma_{\mathrm{oil}}-(0.1\,\mathrm{m})\,\gamma_{\mathrm{Hg}}-(1\,\mathrm{m})\,\gamma_{\mathrm{oil}}=p_1$$

Algebra gives

$$\begin{aligned}\frac{p_2-p_1}{\gamma_{\mathrm{oil}}}&=-(2\,\mathrm{m})-(0.1\,\mathrm{m})+(0.1\,\mathrm{m})\,\frac{\gamma_{\mathrm{Hg}}}{\gamma_{\mathrm{oil}}}+(1\,\mathrm{m})\\&=-(1\,\mathrm{m})+(0.1\,\mathrm{m})\left(\frac{S_{\mathrm{Hg}}}{S_{\mathrm{oil}}}-1\right)\\&=-(1\,\mathrm{m})+(0.1\,\mathrm{m})\left(\frac{13.6}{0.8}-1\right)\\\frac{p_2-p_1}{\gamma_{\mathrm{oil}}}&=0.6\,\mathrm{m}\qquad(2)\end{aligned}$$

Substituting Eq. (2) into (1) gives

$$\begin{aligned}(0.6\,\mathrm{m})&=(-1\,\mathrm{m})+f\frac{L}{D}\frac{V^2}{2g}\\&\text{or}\\f&=1.6\left(\frac{D}{L}\right)\left(\frac{2g}{V^2}\right)\\&=1.6\left(\frac{0.05}{12}\right)\left(\frac{2\times 9.81}{1.2^2}\right)\\&\boxed{f=0.0908}\end{aligned}$$

$$\begin{aligned}f&=\frac{64}{\mathrm{Re}}\\0.0908&=\frac{64\mu}{\rho VD}\\&\text{or}\\\mu&=\frac{0.0908\rho VD}{64}\\&=\frac{0.0908\times(0.8\times 1000)\times 1.2\times 0.05}{64}\\&=\boxed{0.068\quad \mathrm{N\cdot s/m^2}}\end{aligned}$$

$$\begin{aligned}\mathrm{Re}&=\frac{VD\rho}{\mu}\\&=\frac{1.2\times 0.05\times(0.8\times 1000)}{0.068}\\&=706\end{aligned}$$

Thus, flow is $\boxed{\text{laminar.}}$

6.4. Minor Head Losses:

For any pipe system, in addition to the Moody-type friction loss (referred as *Major losses*) computed for the length of pipe, there are additional so-called *minor losses* due to:

1. Pipe entrance or exit
2. Sudden expansion or contraction
3. Bends, elbows, tees, and other fittings
4. Valves, open or partially closed
5. Gradual expansions or contractions

These losses may not be so minor in its effects; e.g., a partially closed valve can cause a greater pressure drop than a long pipe.

Since the flow pattern in fittings and valves is quite complex, the theory is very weak. The losses are commonly measured experimentally and correlated with the pipe flow parameters.

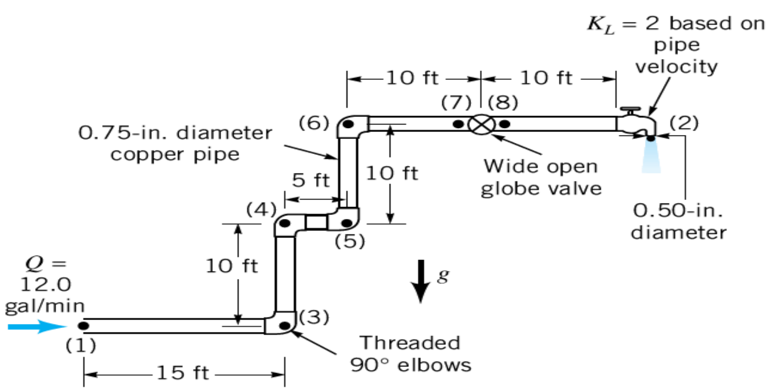
The data, especially for valves, are somewhat dependent upon the particular manufacturer's design, so that the values listed here must be taken as average design estimates

The measured minor loss is usually given as a ratio of the head loss *hm* through the device to the velocity head $\boldsymbol{V^2/(2g)}$ of the associated piping system

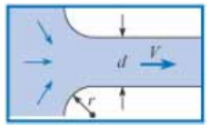
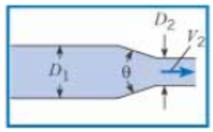
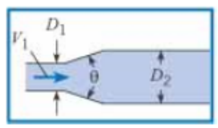
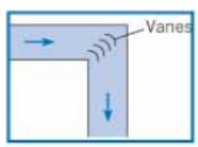
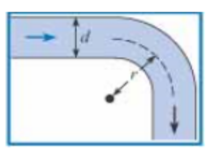
$$\text{Minor loss coefficient, } K = \frac{hm}{V^2/2g}$$

A single pipe system may have many minor losses. Since all are correlated with $V^2/(2g)$, they can be summed into a single total system loss if the pipe has constant diameter

$$hl_{total} = hl_f + \sum hl_m = \frac{V^2}{2g}\left(\frac{fL}{D} + \sum K\right)$$



.5 LOSS COEFFICIENTS FOR VARIOUS TRANSITIONS AND FITTINGS

Description	Sketch	Additional Data		K	Source
Pipe entrance		r/d		K_e	(10)†
$h_L = K_eV^2/2g$		0.0		0.50	
		0.1		0.12	
		>0.2		0.03	
Contraction			K_C	K_C	
$h_L = K_C V_2^2/2g$		D_2/D_1	$\theta = 60°$	$\theta = 180°$	(10)
		0.00	0.08	0.50	
		0.20	0.08	0.49	
		0.40	0.07	0.42	
		0.60	0.06	0.27	
		0.80	0.06	0.20	
		0.90	0.06	0.10	
Expansion			K_E	K_E	
		D_1/D_2	$\theta = 20°$	$\theta = 180°$	(9)
		0.00		1.00	
$h_L = K_E V_1^2/2g$		0.20	0.30	0.87	
		0.40	0.25	0.70	
		0.60	0.15	0.41	
		0.80	0.10	0.15	
90° miter bend		Without vanes	$K_b = 1.1$		(15)
		With vanes	$K_b = 0.2$		(15)
90° smooth bend		r/d			(16) and (9)
		1	$K_b = 0.35$		
		2	0.19		
		4	0.16		
		6	0.21		
		8	0.28		
		10	0.32		
Threaded pipe fittings	Globe valve—wide open			$K_v = 10.0$	(15)
	Angle valve—wide open			$K_v = 5.0$	
	Gate valve—wide open			$K_v = 0.2$	
	Gate valve—half open			$K_v = 5.6$	
	Return bend			$K_b = 2.2$	
	Tee				
	Straight-through flow			$K_t = 0.4$	
	Side-outlet flow			$K_t = 1.8$	
	90° elbow			$K_b = 0.9$	
	45° elbow		$K_b = 0.4$		

Example (10) : ***Find the distance through the pipeline in the figure below for H = 10 m and determine the head loss H for Q = 60 L/s.***

Solution : the energy equation applied between points 1 and 2, including all the losses, can be written as :

$$H_1 + 0 + 0 = \frac{V_2^2}{2g} + 0 + 0 + \frac{1}{2}\frac{V_2^2}{2g} + f\frac{102\text{ m}}{0.15\text{ m}}\frac{V_2^2}{2g} + 2(0.9)\frac{V_2^2}{2g} + 10\frac{V_2^2}{2g}$$

in which the entrance loss coefficient is $\frac{1}{2}$, each elbow is 0.9, and the globe valve is 10. Then

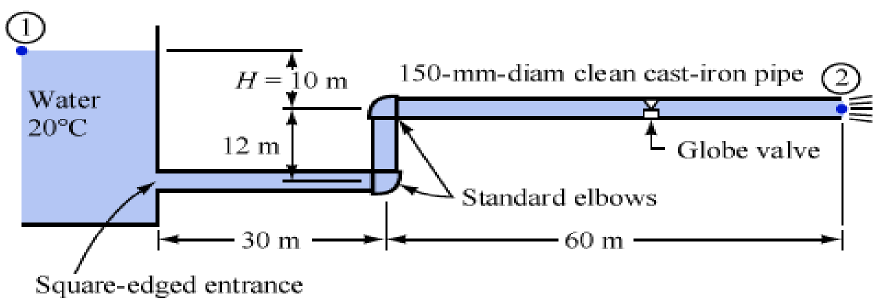
$$H_1 = \frac{V_2^2}{2g}(13.3 + 680f)$$

When the head is given, this problem is solved as the second type of simple pipe problem. If $f = 0.022$,

$$10 = \frac{V_2^2}{2g}[13.3 + 680(0.022)]$$

and $V_2 = 2.63$ m/s. From Appendix C,

$$\nu = 1.01\ \mu\text{ m}^2/\text{s} \qquad \frac{\epsilon}{D} = 0.0017 \qquad \mathbf{R} = \frac{(2.63\text{ m/s})(0.15\text{ m})}{1.01\times10^{-6}\text{ m}^2/\text{s}} = 391{,}000$$



Pipeline with minor losses.

From Fig. 6.21, $f = 0.023$. Repeating the procedure gives $V_2 = 2.60$ m/s, $\mathbf{R} = 380{,}000$, and $f = 0.023$. The discharge is

$$Q = V_2A_2 = (2.60\text{ m/s})\frac{\pi}{4}(0.15\text{ m})^2 = 45.9\text{ L/s}$$

For the second part, with Q known, the solution is straightforward:

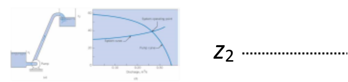
$$V_2 = \frac{Q}{A} = \frac{0.06\text{ m}^3/\text{s}}{(\pi/4)(0.15\text{ m})^2} = 3.40\text{ m/s} \qquad \mathbf{R} = 505{,}000 \qquad f = 0.023$$

and

$$H_1 = \frac{(3.4\text{ m/s})^2}{2(9.806\text{ m/s}^2)}[13.3 + 680(0.023)] = 17.06\text{ m}$$

6.5. Pumps and Turbines:

- Pumps and turbines represent the external source-sink term of energy head
- Apply energy equation from the reservoir water surface to the outlet stream is:



In Ideal Flow:

$$\frac{p_1}{\gamma} + \frac{{v_1}^2}{2g} + z_1 + hp = \frac{p_2}{\gamma} + \frac{{v_2}^2}{2g} + z_2 \quad \ldots . (6.26)$$

For a system with one size of pipe, this simplifies to :

$$hp = (z_2 - z_1) + \frac{{v_2}^2}{2g} \quad \ldots . (6.27)$$

In real Flow with:

$$\frac{p_1}{\gamma} + \frac{{v_1}^2}{2g} + z_1 + hp = \frac{p_2}{\gamma} + \frac{{v_2}^2}{2g} + z_2 + \sum K \frac{{v_2}^2}{2g} + \sum \frac{fL}{D} \frac{{v_2}^2}{2g} \quad \ldots . (6.28)$$

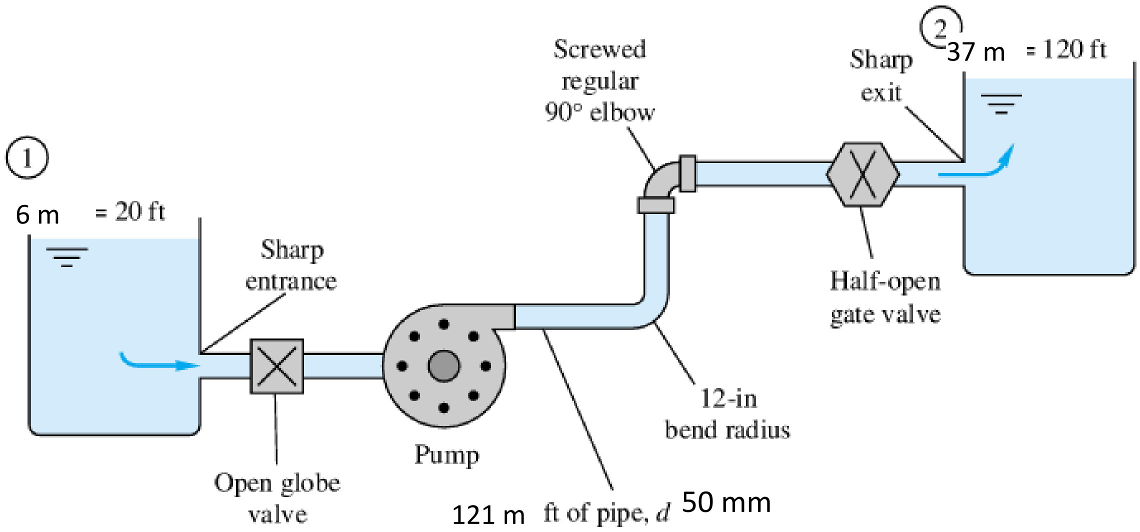
For a system with one size of pipe, this simplifies to :

$$hp = (z_2 - z_1) + \frac{{v_2}^2}{2g} \left(1 + \sum K + \sum \frac{fL}{D}\right) \quad \ldots . (6.29)$$

Pump Power = ρgQh …. (6.30) (hydraulic power of pump, Power out)

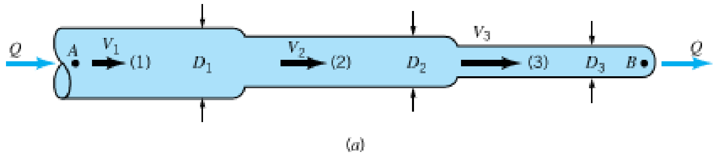
$$Efficiency = \frac{power_{out}}{power_{in}} = \frac{\rho g Q h}{power_{in}} \quad \ldots . (6.31)$$

Example (11) : *Water, $\rho = 1000$ kg/m3 . and $\nu = 1.14 \times 10^{-6}$ m2/s, is pumped between two reservoirs at 6 l/s through 121 m of 50 mm-diameter pipe and several minor losses, as shown in Fig. The roughness ratio is $\varepsilon/d = 0.001$. Compute the pump horsepower required.*

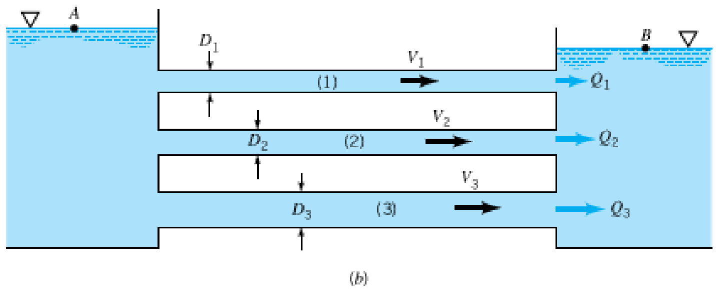


Loss	K
Sharp entranc)	0.5
Open globe valve (2 i	6.9
12-in ber	0.15
Regular 90° elbo	0.95
Half-closed gate valv ›)	2.7
Sharp ex	1.0
	$\sum K = 12.2$

6.6. Pipes Systems :



(a)



(b)

a. Pipes in Series:

$Q_1 = Q_2 = Q_3$ …. (6.32)

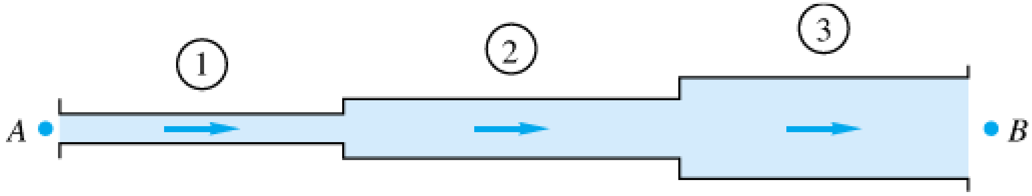
$h_{L\ A\text{-}B} = h_{L1} + h_{L2} + h_{L3}$ …. (6.33)

b. Pipes in Parallel:

$Q_{A\text{-}B} = Q_1 + Q_2 + Q_3$ …. (6.34)

$h_{L1} = h_{L2} = h_{L3}$ …. (6.35)

Example (12): ***Given is a three-pipe series system, as in Fig. The total pressure drop is pA- pB = 150,000 Pa, and the elevation drop is zA - zB = 5 m. The pipe data are in table below. The fluid is water, ρ = 1000 kg/m3 and ν = 1.02 x 10 -6 m2/s. Calculate the flow rate Q in m3/h through the system.***



Pipe	L, m	d, cm	ϵ, mm	ϵ/d
1	100	8	0.24	0.003
2	150	6	0.12	0.002
3	80	4	0.20	0.005

Solution:

The total head loss across the system is

$$\Delta h_{A\to B} = \frac{p_A - p_B}{\rho g} + z_A - z_B = \frac{150{,}000}{1000(9.81)} + 5 \text{ m} = 20.3 \text{ m}$$

From the continuity relation (6.105) the velocities are

$$V_2 = \frac{d_1^2}{d_2^2} V_1 = \frac{16}{9} V_1 \qquad V_3 = \frac{d_1^2}{d_3^2} V_1 = 4V_1$$

and $$\text{Re}_2 = \frac{V_2 d_2}{V_1 d_1} \text{Re}_1 = \frac{4}{3}\text{Re}_1 \qquad \text{Re}_3 = 2 \text{ Re}_1$$

Neglecting minor losses and substituting into Eq. (6.107), we obtain

$$\Delta h_{A\to B} = \frac{V_1^2}{2g}\left[1250 f_1 + 2500\left(\frac{16}{9}\right)^2 f_2 + 2000(4)^2 f_3\right]$$

or $$20.3 \text{ m} = \frac{V_1^2}{2g}(1250 f_1 + 7900 f_2 + 32{,}000 f_3) \qquad (1)$$

This is the form which was hinted at in Eq. (6.108). It seems to be dominated by the third pipe loss $32{,}000 f_3$. Begin by estimating f_1, f_2, and f_3 from the Moody-chart fully rough regime

$$f_1 = 0.0262 \qquad f_2 = 0.0234 \qquad f_3 = 0.0304$$

Substitute in Eq. (1) to find $V_1^2 \approx 2g(20.3)/(33 + 185 + 973)$. The first estimate thus is $V_1 = 0.58$ m/s, from which

$$\text{Re}_1 \approx 45{,}400 \qquad \text{Re}_2 = 60{,}500 \qquad \text{Re}_3 = 90{,}800$$

Hence, from the Moody chart,

$$f_1 = 0.0288 \qquad f_2 = 0.0260 \qquad f_3 = 0.0314$$

Substitution into Eq. (1) gives the better estimate

$$V_1 = 0.565 \text{ m/s} \qquad Q = \tfrac{1}{4}\pi d_1^2 V_1 = 2.84 \times 10^{-3} \text{ m}^3/\text{s}$$

or $$Q_1 = 10.2 \text{ m}^3/\text{h}$$ *Ans.*

A second iteration gives $Q = 10.22$ m^3/h, a negligible change.

Example (13): ***Assume that the same three pipes in Example above are now in parallel with the same total head loss of 20.3 m. Compute the total flow rate Q, neglecting minor losses.***

Pipe	L, m	d, cm	ϵ, mm	ϵ/d
1	100	8	0.24	0.003
2	150	6	0.12	0.002
3	80	4	0.20	0.005

Solution:

$$20.3 \text{ m} = \frac{V_1^2}{2g} 1250 f_1 = \frac{V_2^2}{2g} 2500 f_2 = \frac{V_3^2}{2g} 2000 f_3 \qquad (1)$$

Guess fully rough flow in pipe 1: $f_1 = 0.0262$, $V_1 = 3.49$ m/s; hence $\text{Re}_1 = V_1 d_1/\nu = 273{,}000$. From the Moody chart read $f_1 = 0.0267$; recompute $V_1 = 3.46$ m/s, $Q_1 = 62.5 \text{ m}^3/\text{h}$. [This problem can also be solved from Eq. (6.66).]

Next guess for pipe 2: $f_2 \approx 0.0234$, $V_2 \approx 2.61$ m/s; then $\text{Re}_2 = 153{,}000$, and hence $f_2 = 0.0246$, $V_2 = 2.55$ m/s, $Q_2 = 25.9 \text{ m}^3/\text{h}$.

Finally guess for pipe 3: $f_3 \approx 0.0304$, $V_3 \approx 2.56$ m/s; then $\text{Re}_3 = 100{,}000$, and hence $f_3 = 0.0313$, $V_3 = 2.52$ m/s, $Q_3 = 11.4 \text{ m}^3/\text{h}$.

This is satisfactory convergence. The total flow rate is

$$Q = Q_1 + Q_2 + Q_3 = 62.5 + 25.9 + 11.4 = 99.8 \text{ m}^3/\text{h} \qquad \textit{Ans.}$$